

Linear and Quadratic Simultaneous Equations

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Solve the simultaneous equations

$$y = x^2 - 4$$

$$y = 3x$$

Both equations say what y is, so put them equal:

$$x^2 - 4 = 3x$$

Subtract $3x$ from both sides: $x^2 - 3x - 4 = 0$

Factorise: $(x - 4)(x + 1) = 0$

So $x = 4$ or $x = -1$

Find each y using $y = 3x$:

When $x = 4$: $y = 12$

When $x = -1$: $y = -3$

Answer: $x = 4, y = 12$ and $x = -1, y = -3$

Tip: Give your answers in matching pairs: $x = 4$ goes with $y = 12$, and $x = -1$ goes with $y = -3$.

Marks: 1 mark for putting the two expressions for y equal. 1 mark for $(x - 4)(x + 1)$. 1 mark for both x values. 1 mark for both y values.

2 (5 marks)

Solve algebraically the simultaneous equations

$$x^2 + y^2 = 13$$

$$y - x = 1$$

Rearrange the linear equation: $y = x + 1$

Substitute into $x^2 + y^2 = 13$:

$$x^2 + (x + 1)^2 = 13$$

Expand: $x^2 + x^2 + 2x + 1 = 13$

$$2x^2 + 2x - 12 = 0$$

Divide every term by 2: $x^2 + x - 6 = 0$

Factorise: $(x + 3)(x - 2) = 0$

So $x = 2$ or $x = -3$

Find each y using $y = x + 1$:

When $x = 2$: $y = 3$

When $x = -3$: $y = -2$

Answer: $x = 2, y = 3$ and $x = -3, y = -2$

Marks: 1 mark for rearranging and substituting. 1 mark for expanding to a three-term quadratic. 1 mark for $(x + 3)(x - 2)$. 1 mark for both x values. 1 mark for both pairs.

3 (5 marks)

Solve algebraically the simultaneous equations

$$y = 2x^2 - 5x + 1$$

$$y = x - 3$$

Put the two expressions for y equal:

$$2x^2 - 5x + 1 = x - 3$$

Subtract x and add 3 on both sides: $2x^2 - 6x + 4 = 0$

Divide every term by 2: $x^2 - 3x + 2 = 0$

Factorise: $(x - 1)(x - 2) = 0$

So $x = 1$ or $x = 2$

Find each y using $y = x - 3$:

When $x = 1$: $y = -2$

When $x = 2$: $y = -1$

Answer: $x = 1, y = -2$ and $x = 2, y = -1$

Marks: 1 mark for putting the expressions equal. 1 mark for $x^2 - 3x + 2 = 0$. 1 mark for $(x - 1)(x - 2)$. 1 mark for both x values. 1 mark for both pairs.

4 (5 marks)

Solve algebraically the simultaneous equations

$$2x^2 + y^2 = 22$$

$$x + y = 1$$

Rearrange the linear equation: $y = 1 - x$

Substitute into $2x^2 + y^2 = 22$:

$$2x^2 + (1 - x)^2 = 22$$

Expand: $(1 - x)^2 = 1 - 2x + x^2$

$$2x^2 + 1 - 2x + x^2 = 22$$

$$3x^2 - 2x - 21 = 0$$

Factorise: $(3x + 7)(x - 3) = 0$

So $x = 3$ or $x = -\frac{7}{3}$

Find each y using $y = 1 - x$:

When $x = 3$: $y = 1 - 3 = -2$

When $x = -\frac{7}{3}$: $y = 1 + \frac{7}{3} = \frac{10}{3}$

Answer: $x = 3, y = -2$ and $x = -\frac{7}{3}, y = \frac{10}{3}$

Tip: $(1 - x)^2$ has three terms: $1 - 2x + x^2$. Writing $1 + x^2$ or $1 - x^2$ loses marks.

Marks: 1 mark for rearranging and substituting. 1 mark for $3x^2 - 2x - 21 = 0$. 1 mark for $(3x + 7)(x - 3)$. 1 mark for both x values. 1 mark for both pairs.

5 (5 marks)

Here are the equations of a curve and a straight line.

curve: $y = x^2 - 3x + 7$

line: $y = 3x - 2$

Show algebraically that the line is a tangent to the curve.

Find the coordinates of the point where the line touches the curve.

Where the line meets the curve, the y values are equal:

$$x^2 - 3x + 7 = 3x - 2$$

Subtract $3x$ and add 2 on both sides: $x^2 - 6x + 9 = 0$

Factorise: $(x - 3)(x - 3) = 0$, which is $(x - 3)^2 = 0$

This has only one solution, $x = 3$ (a repeated root).

So the line and the curve meet at exactly one point, which means the line is a tangent to the curve.

(Or: $b^2 - 4ac = (-6)^2 - 4 \times 1 \times 9 = 36 - 36 = 0$, so there is exactly one solution.)

Find y using $y = 3x - 2$: $y = 3 \times 3 - 2 = 7$

Answer: (3, 7)

Tip: Finding $x = 3$ is not enough on its own. Say clearly that the equation has only **one** solution, so the line meets the curve at exactly one point.

Marks: 1 mark for putting the expressions equal. 1 mark for $x^2 - 6x + 9 = 0$. 1 mark for $(x - 3)^2 = 0$ or $b^2 - 4ac = 0$. 1 mark for saying there is only one solution, so the line is a tangent. 1 mark for (3, 7).

6 (4 marks)

Solve algebraically the simultaneous equations

$$x^2 + y^2 = 15$$

$$x + 2y = 3$$

Give your solutions correct to 3 significant figures.

Rearrange the linear equation to make x the subject: $x = 3 - 2y$

Substitute into $x^2 + y^2 = 15$:

$$(3 - 2y)^2 + y^2 = 15$$

$$\text{Expand: } 9 - 12y + 4y^2 + y^2 = 15$$

$$\text{Rearrange so one side is 0: } 5y^2 - 12y - 6 = 0$$

This does not factorise, so use the quadratic formula ($a = 5$, $b = -12$, $c = -6$):

$$y = \frac{12 \pm \sqrt{(-12)^2 - 4 \times 5 \times (-6)}}{2 \times 5} = \frac{12 \pm \sqrt{264}}{10}$$

$$y = 2.8248 \dots \text{ or } y = -0.4248 \dots$$

Find each x using $x = 3 - 2y$:

$$\text{When } y = 2.8248 \dots: x = -2.6496 \dots$$

$$\text{When } y = -0.4248 \dots: x = 3.8496 \dots$$

Answer: $x = -2.65, y = 2.82$ and $x = 3.85, y = -0.425$

Tip: Here it is easier to make x the subject, because x has no number in front of it. Use the unrounded y values (from your calculator) to work out x , then round all four answers to 3 significant figures.

Marks: 1 mark for substituting $x = 3 - 2y$. 1 mark for $5y^2 - 12y - 6 = 0$. 1 mark for correct use of the formula. 1 mark for all four values.