

Inverse Functions

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

$$f(x) = \frac{3x - 4}{2}$$

Find $f^{-1}(x)$

Write $y = \frac{3x - 4}{2}$ and rearrange to make x the subject.

Multiply both sides by 2: $2y = 3x - 4$

Add 4 to both sides: $2y + 4 = 3x$

Divide both sides by 3: $x = \frac{2y + 4}{3}$

Now write the inverse in terms of x .

Answer: $f^{-1}(x) = \frac{2x + 4}{3}$

Tip: Your final answer must be in terms of x , e.g. $f^{-1}(x) = \frac{2x + 4}{3}$. Leaving it in terms of y loses the last mark. And $f^{-1}(x)$ is not $\frac{1}{f(x)}$.

Marks: 1 mark for a correct first step, e.g. $2y = 3x - 4$. 1 mark for the correct answer.

2 (3 marks)

$$f(x) = x^3 - 1$$

(a) Find $f^{-1}(x)$

Write $y = x^3 - 1$

Add 1 to both sides: $y + 1 = x^3$

Cube root both sides: $x = \sqrt[3]{y+1}$

Now write the inverse in terms of x .

Answer: $f^{-1}(x) = \sqrt[3]{x+1}$

Tip: Make sure the cube root sign covers the whole of $x + 1$. $\sqrt[3]{x} + 1$ is wrong.

Marks: 1 mark for $x^3 = y + 1$. 1 mark for the correct answer.

(b) Find $f^{-1}(26)$

Substitute $x = 26$ into your answer to part (a).

$$f^{-1}(26) = \sqrt[3]{26+1} = \sqrt[3]{27} = 3$$

Check: $f(3) = 27 - 1 = 26$ ✓

Answer: 3

3 (3 marks)

$$g(x) = \frac{x}{x+5}$$

Find $g^{-1}(x)$

Write $y = \frac{x}{x+5}$

Multiply both sides by $(x+5)$: $y(x+5) = x$

Expand the bracket: $xy + 5y = x$

Get the x terms on one side: $xy - x = -5y$

Factorise out x : $x(y-1) = -5y$

Divide by $(y-1)$: $x = \frac{-5y}{y-1}$

Multiply top and bottom by -1 to tidy it: $x = \frac{5y}{1-y}$

Now write the inverse in terms of x .

Answer: $g^{-1}(x) = \frac{5x}{1-x}$ (or $\frac{-5x}{x-1}$)

Marks: 1 mark for multiplying out, $xy + 5y = x$. 1 mark for collecting the x terms and factorising, $x(y - 1) = -5y$. 1 mark for the correct answer.

4 (3 marks)

$f(x) = 3x + 5$ and $g(x) = x - 7$

Solve $f^{-1}(x) = g(x)$

First find the inverse of f .

$y = 3x + 5$, so $y - 5 = 3x$

So $f^{-1}(x) = \frac{x-5}{3}$

Now solve $\frac{x-5}{3} = x - 7$

Multiply both sides by 3: $x - 5 = 3x - 21$

Subtract x and add 21 to both sides: $16 = 2x$

Divide by 2: $x = 8$

Check: $f^{-1}(8) = \frac{3}{3} = 1$ and $g(8) = 1$ ✓

Answer: $x = 8$

Tip: When you multiply $x - 7$ by 3, multiply both terms: $3x - 21$, not $3x - 7$.

Marks: 1 mark for $f^{-1}(x) = \frac{x-5}{3}$. 1 mark for an equation without fractions, e.g. $x - 5 = 3x - 21$. 1 mark for $x = 8$.

5 (5 marks)

The functions f and g are such that

$$f(x) = x^2 - 3x \quad \text{and} \quad g(x) = \frac{x+6}{2}$$

- (a) Find $g^{-1}(x)$

Write $y = \frac{x+6}{2}$

Multiply both sides by 2: $2y = x + 6$

Subtract 6: $x = 2y - 6$

Now write the inverse in terms of x .

Answer: $g^{-1}(x) = 2x - 6$

Marks: 1 mark for $2y = x + 6$. 1 mark for the correct answer.

- (b) Given that $gf(k) = 5$
find the possible values of k .

$gf(k)$ means: put k into f first, then put the answer into g .

$$f(k) = k^2 - 3k$$

So $gf(k) = \frac{k^2 - 3k + 6}{2}$

Solve $\frac{k^2 - 3k + 6}{2} = 5$

Multiply by 2: $k^2 - 3k + 6 = 10$

Rearrange: $k^2 - 3k - 4 = 0$

Factorise: $(k - 4)(k + 1) = 0$

(Or: $f(k) = g^{-1}(5) = 2 \times 5 - 6 = 4$, so $k^2 - 3k = 4$.)

Answer: $k = 4$ or $k = -1$

Tip: $gf(k)$ uses f first. There are two answers because the equation is a quadratic.

Marks: 1 mark for $\frac{k^2-3k+6}{2} = 5$ or $k^2 - 3k = 4$. 1 mark for $k^2 - 3k - 4 = 0$. 1 mark for both values.

6 (4 marks)

The functions f and g are such that

$$f(x) = 10 - 3x \quad \text{and} \quad g(x) = \sqrt{x + 4}$$

(a) Find $fg(5)$

$fg(5)$ means: put 5 into g first, then put the answer into f .

$$g(5) = \sqrt{5 + 4} = \sqrt{9} = 3$$

$$f(3) = 10 - 3 \times 3 = 1$$

Answer: 1

Marks: 1 mark for $g(5) = 3$. 1 mark for 1.

(b) Find $f^{-1}(x)$

Write $y = 10 - 3x$

Add $3x$ to both sides: $y + 3x = 10$

Subtract y : $3x = 10 - y$

Divide by 3: $x = \frac{10 - y}{3}$

Now write the inverse in terms of x .

Answer: $f^{-1}(x) = \frac{10 - x}{3}$

Tip: Take care with the sign: the answer is $\frac{10 - x}{3}$, not $\frac{x - 10}{3}$. Check: $f(2) = 4$ and $\frac{10 - 4}{3} = 2$.

Marks: 1 mark for $3x = 10 - y$ or equivalent. 1 mark for the correct answer.

7 (3 marks)

$$f(x) = \frac{2}{x}$$

$$g(x) = x + 3$$

$$h(x) = gf(x)$$

Find $h^{-1}(x)$

First find $h(x)$ by putting $f(x)$ into g :

$$h(x) = g\left(\frac{2}{x}\right) = \frac{2}{x} + 3$$

Write $y = \frac{2}{x} + 3$

Subtract 3: $y - 3 = \frac{2}{x}$

Multiply by x : $x(y - 3) = 2$

Divide by $(y - 3)$: $x = \frac{2}{y - 3}$

Now write the inverse in terms of x .

Answer: $h^{-1}(x) = \frac{2}{x - 3}$

Tip: $gf(x)$ means put $f(x)$ into g : $\frac{2}{x} + 3$, not $\frac{2}{x + 3}$.

Marks: 1 mark for $h(x) = \frac{2}{x} + 3$. 1 mark for $y - 3 = \frac{2}{x}$ or $x(y - 3) = 2$. 1 mark for the correct answer.

8 (4 marks)

The functions p and q are such that

$$p(x) = 4x - 3 \quad \text{and} \quad q(x) = \frac{x}{x + 1}$$

(a) Find $p^{-1}(9)$

$p^{-1}(9)$ is the value of x that makes $p(x) = 9$.

$$4x - 3 = 9, \text{ so } 4x = 12, \text{ so } x = 3$$

Answer: 3

(b) Find $p^{-1}q(x)$

Give your answer as a single fraction in its simplest form.

First find $p^{-1}(x)$.

$$y = 4x - 3, \text{ so } y + 3 = 4x$$

$$\text{So } p^{-1}(x) = \frac{x+3}{4}$$

$p^{-1}q(x)$ means put $q(x)$ into p^{-1} :

$$p^{-1}\left(\frac{x}{x+1}\right) = \frac{\frac{x}{x+1} + 3}{4}$$

$$\text{Write the top as one fraction: } \frac{x}{x+1} + 3 = \frac{x+3(x+1)}{x+1} = \frac{4x+3}{x+1}$$

$$\text{Divide by 4: } \frac{4x+3}{4(x+1)}$$

$$\textbf{Answer: } p^{-1}q(x) = \frac{4x+3}{4(x+1)} \text{ or } \frac{4x+3}{4x+4}$$

Tip: Find p^{-1} first, then put $q(x)$ into it. Do not leave a fraction inside a fraction: combine the top into one fraction first.

Marks: 1 mark for $p^{-1}(x) = \frac{x+3}{4}$. 1 mark for $\frac{\frac{x}{x+1} + 3}{4}$. 1 mark for the correct single fraction.
