

Inequalities and Regions on Graphs

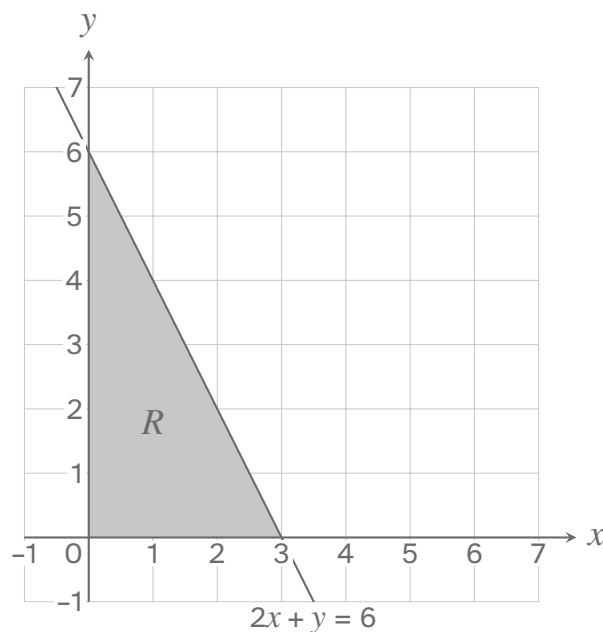
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

The line with equation $2x + y = 6$ is drawn on the grid.



- (a) On the grid, show by shading the region that satisfies all three of these inequalities.

$$x \geq 0 \quad y \geq 0 \quad 2x + y \leq 6$$

Label the region R .

$x \geq 0$: on or to the right of the y -axis.

$y \geq 0$: on or above the x -axis.

$2x + y \leq 6$: on or below the line $2x + y = 6$.

R is the triangle shaded in the diagram above.

Answer: See diagram

Marks: 1 mark for the region below $2x + y = 6$. 1 mark for the correct region labelled R .

- (b) Sam says that the point $(2, 2)$ is in the region R .

Is Sam correct?

You must give a reason for your answer.

Substitute $x = 2, y = 2$: $2 \times 2 + 2 = 6$

So $(2, 2)$ lies on the line $2x + y = 6$.

The inequality is $\leq$, so points on the line are included.

$(2, 2)$ also fits $x \geq 0$ and $y \geq 0$.

Answer: Yes: $2 \times 2 + 2 = 6 \leq 6$

Tip: A point on a solid boundary line **is** in the region. Many students wrongly say it is not.

2 (4 marks)

On the grid, show by shading the region that satisfies all three of these inequalities.
Label the region R .

$$y < 5 \quad y > 2x - 1 \quad 2x + y > 3$$

All three inequalities are strict ($<$ or $>$), so draw dashed lines.

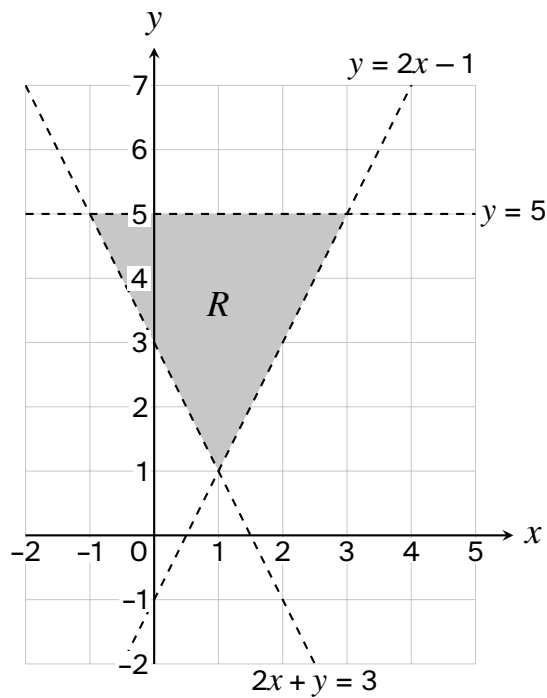
$y = 5$ is a horizontal line.

$y = 2x - 1$ goes through $(0, -1)$ with gradient 2.

$2x + y = 3$ goes through $(0, 3)$ and $(1.5, 0)$.

R is below $y = 5$, above (to the left of) $y = 2x - 1$, and above $2x + y = 3$.

R is the triangle with vertices $(-1, 5)$, $(3, 5)$ and $(1, 1)$.



Check with $(1, 3)$: $3 < 5$ ✓ $3 > 2 \times 1 - 1 = 1$ ✓ $2 \times 1 + 3 = 5 > 3$ ✓

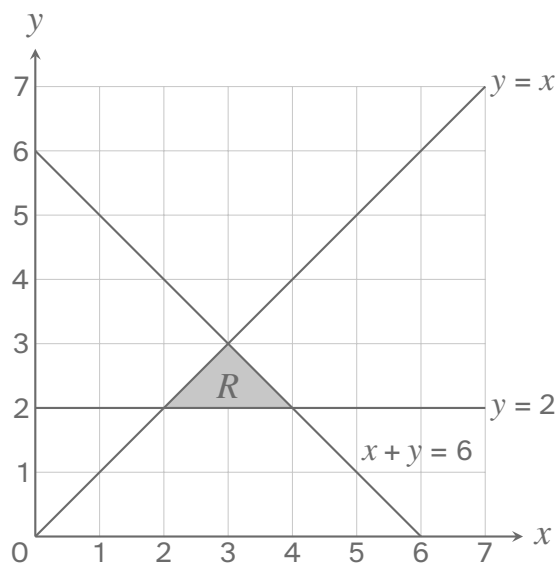
Answer: Triangle with vertices $(-1, 5)$, $(3, 5)$, $(1, 1)$

Tip: To draw $2x + y = 3$, find two points on it: $x = 0$ gives $y = 3$, and $x = 2$ gives $y = -1$. Then check which side you need with a test point such as $(0, 0)$: $0 > 3$ is false, so R is on the other side.

Marks: 1 mark for any one correct line. 1 mark for all three lines. 1 mark for the correct region. 1 mark for labelling it R .

3 (3 marks)

The region R is shown shaded on the grid.



Write down the three inequalities that define the region R .

R is above the line $y = 2$: $y \geq 2$

R is below (to the right of) the line $y = x$: $y \leq x$

R is below the line $x + y = 6$: $x + y \leq 6$

All three lines are solid, so use $\leq$ and $\geq$.

Answer: $y \geq 2, y \leq x, x + y \leq 6$

Marks: 1 mark for each correct inequality.

4 (3 marks)

x and y are integers.

$$x \geq 1 \quad y > x \quad x + y < 7$$

How many pairs (x, y) satisfy all three inequalities?

Go through the values of x in order.

$x = 1$: $y > 1$ and $y < 6$, so $y = 2, 3, 4, 5$ (4 pairs)

$x = 2$: $y > 2$ and $y < 5$, so $y = 3, 4$ (2 pairs)

$x = 3$: $y > 3$ and $y < 4$, so there are no integers.

Larger values of x give no pairs either.

Total: $4 + 2 = 6$ pairs

Answer: 6

Marks: 1 mark for listing pairs in a systematic way. 1 mark for at least 4 correct pairs. 1 mark for 6.

5 (3 marks)

On the grid, show by shading the region that satisfies all three of these inequalities.

$$x \geq 2 \quad y \leq 8 - 2x \quad y \geq 2x - 8$$

Label the region R .

All three inequalities use $\leq$ or $\geq$, so all the lines are solid.

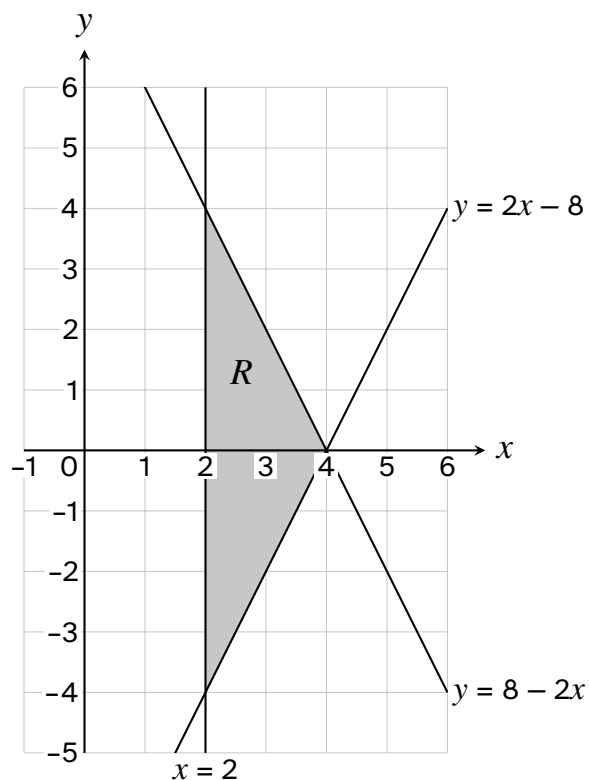
$x = 2$ is a vertical line.

$y = 8 - 2x$ goes through $(0, 8)$ and $(4, 0)$. It has gradient -2 .

$y = 2x - 8$ goes through $(0, -8)$ and $(4, 0)$. It has gradient 2 .

R is on or right of $x = 2$, on or below $y = 8 - 2x$, and on or above $y = 2x - 8$.

R is the triangle with vertices $(2, 4)$, $(2, -4)$ and $(4, 0)$.



Check with (3, 0): $3 \geq 2$ ✓ $0 \leq 8 - 6 = 2$ ✓ $0 \geq 6 - 8 = -2$ ✓

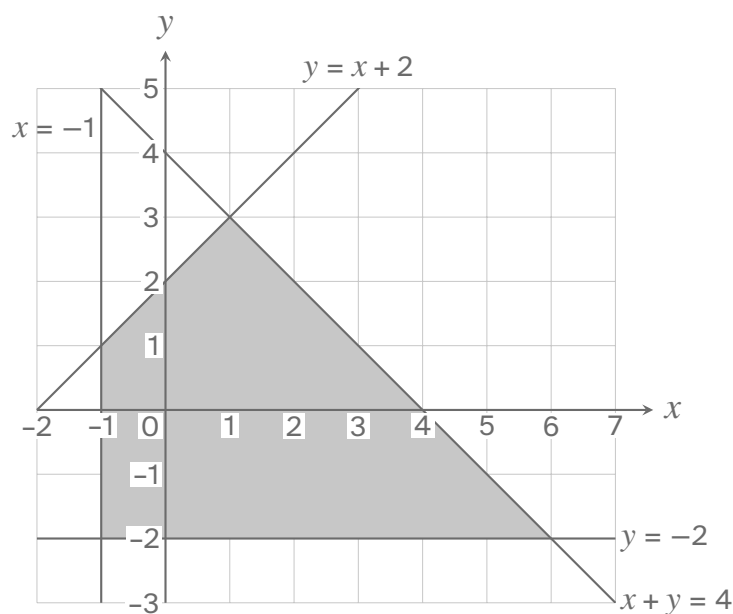
Answer: Triangle with vertices (2, 4), (2, -4), (4, 0)

Tip: $y = 8 - 2x$ has gradient -2 , so it slopes downwards. A common mistake is to draw it with gradient $+2$.

Marks: 2 marks for all three lines correct (1 mark for two lines). 1 mark for the correct region labelled R .

6 (4 marks)

The diagram shows a shaded region.



Find the **four** inequalities that define the shaded region.

The region is to the right of $x = -1$: $x \geq -1$

It is above $y = -2$: $y \geq -2$

It is below $y = x + 2$: $y \leq x + 2$

It is below $x + y = 4$: $x + y \leq 4$

All the lines are solid, so none of the inequalities are strict.

Answer: $x \geq -1$, $y \geq -2$, $y \leq x + 2$, $x + y \leq 4$

Marks: 1 mark for each correct inequality.

7 (5 marks)

(a) On the grid, show by shading the region that satisfies all of these inequalities.

$$x \geq -2 \quad x < 3 \quad y > -1 \quad 2y \leq x + 6$$

Label the region R .

$x \geq -2$ and $2y \leq x + 6$ use $\leq$ or $\geq$: draw these lines solid.

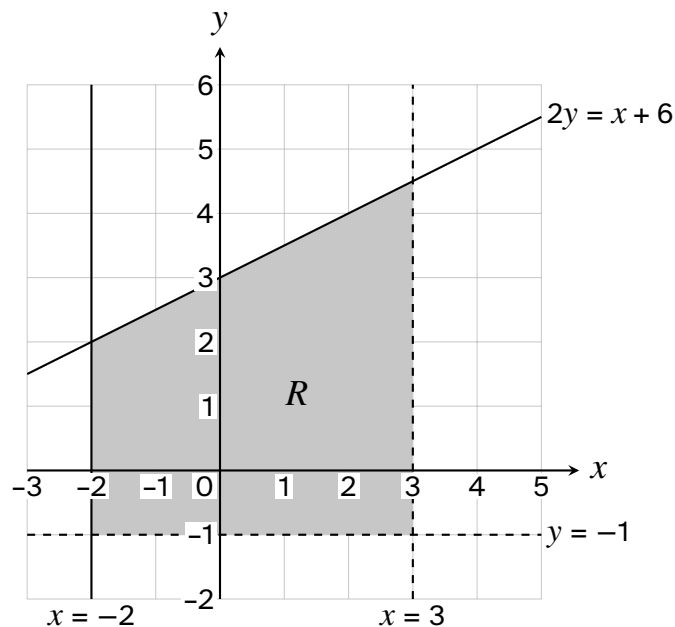
$x < 3$ and $y > -1$ are strict: draw these lines dashed.

$x = -2$ and $x = 3$ are vertical. $y = -1$ is horizontal.

$2y = x + 6$ is the same as $y = \frac{1}{2}x + 3$. It goes through $(-2, 2)$, $(0, 3)$ and $(2, 4)$.

R is between $x = -2$ and $x = 3$, above $y = -1$, and on or below $2y = x + 6$.

R has vertices $(-2, -1)$, $(3, -1)$, $(3, 4.5)$ and $(-2, 2)$.



Answer: Region with vertices $(-2, -1)$, $(3, -1)$, $(3, 4.5)$, $(-2, 2)$

Tip: Rearrange $2y = x + 6$ as $y = \frac{1}{2}x + 3$ before you draw it. The gradient is $\frac{1}{2}$, not 1.

Marks: 1 mark for any two correct lines. 1 mark for all four lines. 1 mark for the correct region. 1 mark for labelling it R .

- (b) The point $(a, 4)$ is in the region R .

a is an integer.

Find the value of a .

Points in R with $y = 4$ must satisfy $2 \times 4 \leq x + 6$, so $x \geq 2$.

They must also satisfy $x < 3$.

So $2 \leq a < 3$.

The only integer is $a = 2$ (the point $(2, 4)$ is on the solid line, so it is included).

Answer: 2

Tip: $x < 3$ is strict, so $a = 3$ is not allowed: the point $(3, 4)$ is on the dashed line.
