

Graph Transformations

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

- (a) The graph of $y = x^2$ is translated 4 units up.
Write down the equation of the translated graph.

Moving up 4 adds 4 to every y-value: $y = f(x) + 4$

Answer: $y = x^2 + 4$

- (b) The graph of $y = x^2$ is translated 3 units to the left.
Write down the equation of the translated graph.

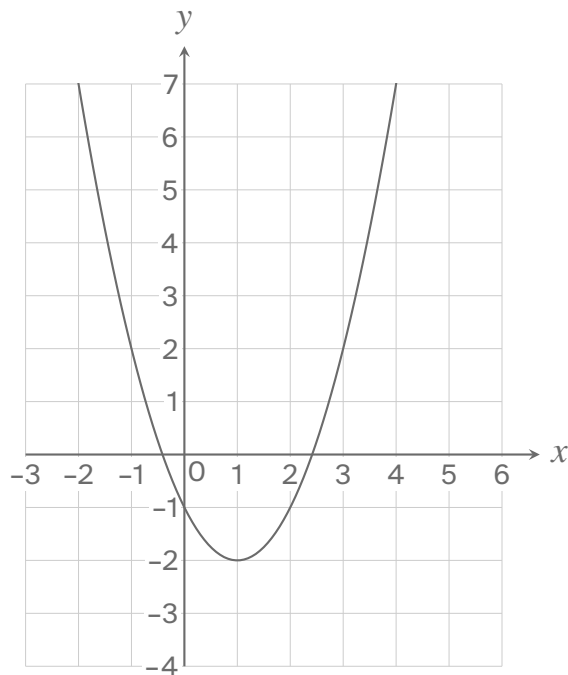
Moving left 3 is $f(x + 3)$, so replace x with $(x + 3)$.

Answer: $y = (x + 3)^2$

Tip: A change inside the bracket moves the graph the opposite way to its sign: $(x + 3)$ moves it 3 to the **left**. $y = x^2 + 3$ or $y = (x - 3)^2$ are wrong.

2 (4 marks)

The graph of $y = f(x)$ is shown on the grid.



- (a) Write down the coordinates of the turning point of the graph of $y = f(x)$

Read from the graph: the lowest point of the curve is at $x = 1$, $y = -2$.

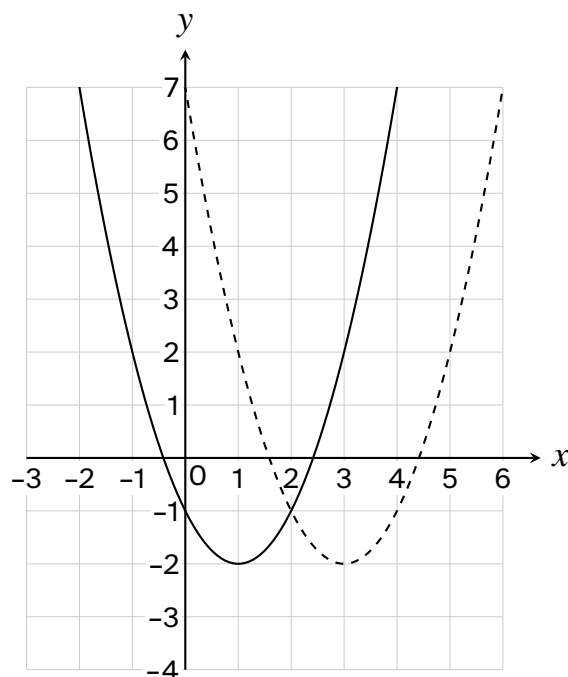
Answer: (1, -2)

- (b) On the same grid, sketch the graph of $y = f(x - 2)$

$f(x - 2)$ moves the graph 2 to the **right**.

The turning point moves from (1, -2) to (3, -2).

The new graph is shown dashed:



Answer: The same curve moved 2 to the right, with turning point $(3, -2)$

Tip: Students often moved the graph the wrong way. $f(x - 2)$ moves it right; $f(x + 2)$ moves it left.

Marks: 2 marks for a correct sketch. 1 mark for the correct shape moved sideways, or a curve through $(3, -2)$.

- (c) The graph of $y = f(x) + k$ touches the x -axis at exactly one point.
Find the value of k .

$f(x) + k$ moves the graph up by k .

The turning point $(1, -2)$ is the lowest point, so it must move up onto the x -axis.

From $y = -2$ to $y = 0$ is up 2, so $k = 2$.

Answer: $k = 2$

Tip: The graph touches the x -axis at one point when its turning point is on the x -axis.

3 (2 marks)

- (a) The graph of $y = \cos x^\circ$ is translated to give the graph of $y = \cos x^\circ + 2$
Describe the transformation.

Adding 2 outside the function moves the graph up 2.

This is a translation by the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

Answer: Translation by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

- (b) The graph of $y = \cos x^\circ$ is translated to give the graph of $y = \cos(x - 30)^\circ$
Describe the transformation.

$\cos(x - 30)^\circ$ is $f(x - 30)$, which moves the graph 30 to the right.

This is a translation by the vector $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$.

Answer: Translation by $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$

4 (3 marks)

The point $A(3, 4)$ lies on the curve $y = g(x)$

Write down the coordinates of the image of A on the curve with equation

- (i) $y = g(-x)$ (ii) $y = -g(x)$ (iii) $y = g(x + 3)$

(i) $g(-x)$ is a reflection in the y -axis, so the x -coordinate changes sign: $(-3, 4)$

(ii) $-g(x)$ is a reflection in the x -axis, so the y -coordinate changes sign: $(3, -4)$

(iii) $g(x + 3)$ is a translation 3 to the left, so subtract 3 from the x -coordinate: $(0, 4)$

Answer: (i) $(-3, 4)$ (ii) $(3, -4)$ (iii) $(0, 4)$

Tip: Changes **inside** the bracket affect the x -coordinate. Changes **outside** affect the y -coordinate.

Marks: 1 mark for each correct point.

5 (4 marks)

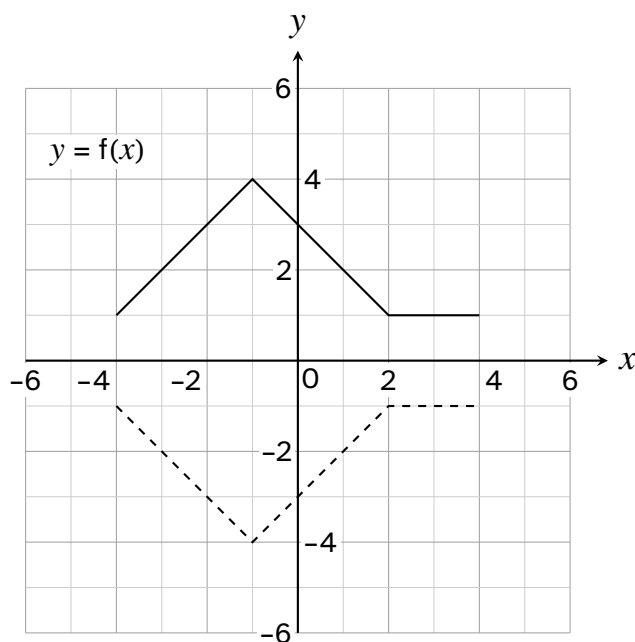
The graph of $y = f(x)$ is shown on each grid.

(a) On the grid, sketch the graph of $y = -f(x)$

$-f(x)$ is a reflection in the x -axis: each y -coordinate changes sign.

The corners move to $(-4, -1)$, $(-1, -4)$, $(2, -1)$, $(4, -1)$

Join them up (shown dashed):



Answer: The graph reflected in the x -axis

Marks: 2 marks for a correct graph. 1 mark for three correct points.

(b) On the grid, sketch the graph of $y = f(x + 1) - 3$

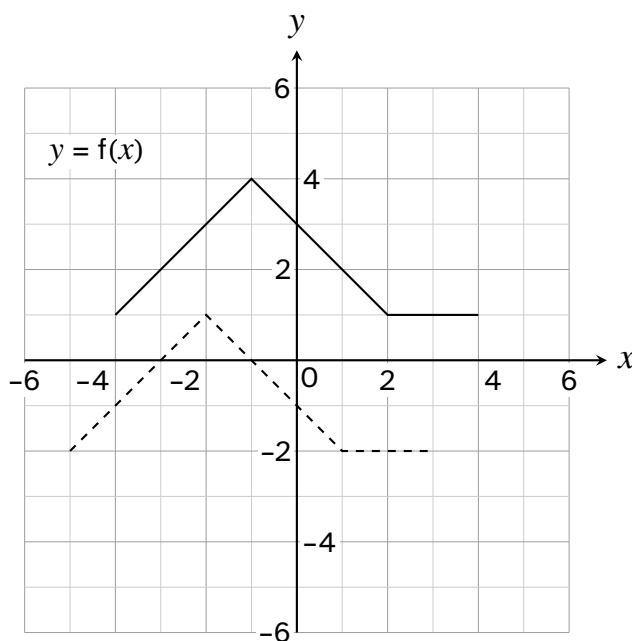
$f(x + 1)$ moves the graph 1 to the **left**.

Then -3 moves it down 3.

This is a translation by $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$.

The corners move to $(-5, -2)$, $(-2, 1)$, $(1, -2)$, $(3, -2)$

Join them up (shown dashed):



Answer: The graph moved 1 to the left and 3 down

Tip: $(x + 1)$ inside the bracket moves the graph 1 to the **left**, not the right. Move each corner point, then join them up.

Marks: 2 marks for a correct graph. 1 mark for a translation 1 to the left or 3 down.

6 (2 marks)

The curve C has equation $y = x^2 + 2x - 3$

The curve C is translated to give the curve D with equation $y = x^2 - 6x + 5$

Find the vector of the translation.

Complete the square to find the turning point of each curve.

$C: x^2 + 2x - 3 = (x + 1)^2 - 1 - 3 = (x + 1)^2 - 4$, so the turning point is $(-1, -4)$

$D: x^2 - 6x + 5 = (x - 3)^2 - 9 + 5 = (x - 3)^2 - 4$, so the turning point is $(3, -4)$

From $(-1, -4)$ to $(3, -4)$: 4 to the right and 0 up or down.

(Or factorise: $x^2 + 2x - 3 = (x + 3)(x - 1)$, so C crosses the x -axis at -3 and 1 ;

$x^2 - 6x + 5 = (x - 1)(x - 5)$, so D crosses at 1 and 5 . Both turning points are at $y = -4$, so the curve has moved 4 to the right only.)

Answer: $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

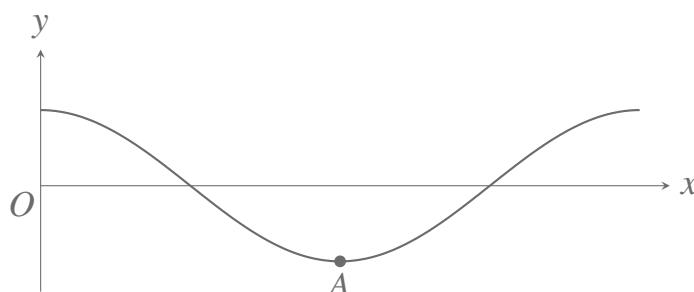
Tip: Compare the same point on both curves (for example the turning points). Do not just compare the numbers in the two equations.

Marks: 1 mark for the roots or the turning point of both curves. 1 mark for the correct vector.

7 (2 marks)

Here is a sketch of the graph of $y = \cos x^\circ$ for $0 \leq x \leq 360$

The point marked A is the minimum point on the graph.



The graph of $y = \cos x^\circ$ is translated by the vector $\begin{pmatrix} 30 \\ 2 \end{pmatrix}$

(a) Write down the coordinates of the image of A .

A is the minimum point of $y = \cos x^\circ$, which is $(180, -1)$.

The vector moves every point 30 to the right and 2 up.

$$(180, -1) \rightarrow (180 + 30, -1 + 2) = (210, 1)$$

Answer: $(210, 1)$

- (b) Write down an equation of the translated graph.

30 to the right: replace x with $(x - 30)$, giving $\cos(x - 30)^\circ$

2 up: add 2

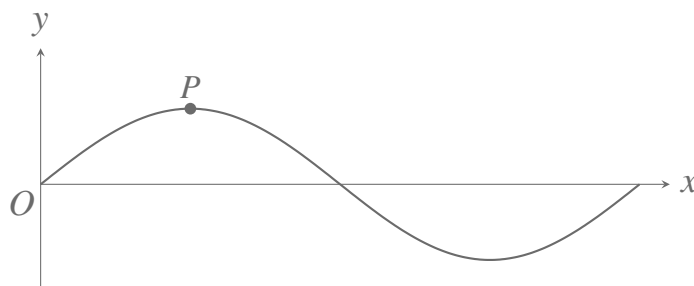
Answer: $y = \cos(x - 30)^\circ + 2$

Tip: Moving right uses $(x - 30)$ inside the bracket. $\cos(x + 30)$ would move the graph to the left.

8 (2 marks)

Here is a sketch of the graph of $y = \sin x^\circ$ for $0 \leq x \leq 360$

The point marked P is a turning point on the graph.



The graph of $y = \sin x^\circ$ is reflected in the x -axis and then translated 3 units up, to give the graph of $y = g(x)$

- (a) Find an expression for $g(x)$ in terms of x .

Reflecting in the x -axis changes $\sin x^\circ$ to $-\sin x^\circ$.

Moving up 3 adds 3.

$$g(x) = -\sin x^\circ + 3$$

Answer: $g(x) = 3 - \sin x^\circ$

Tip: A reflection in the x -axis puts a minus sign in front of the whole function: $-\sin x$. $\sin(-x)$ would be a reflection in the y -axis.

- (b) Find the coordinates of the image of P on the graph of $y = g(x)$

P is the maximum point of $y = \sin x^\circ$, which is $(90, 1)$.

Reflect in the x -axis: $(90, 1) \rightarrow (90, -1)$

Move up 3: $(90, -1) \rightarrow (90, 2)$

Check: $g(90) = 3 - \sin 90^\circ = 3 - 1 = 2 \checkmark$

Answer: $(90, 2)$

Tip: Do the two steps in order: reflect first, then move up. Moving up first and then reflecting gives $(90, -4)$, which is wrong.
