

Gradients and Areas Under Graphs

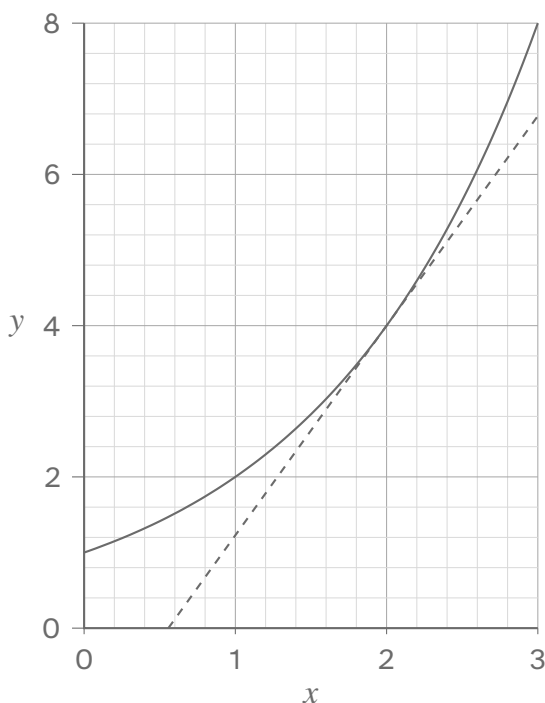
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (7 marks)

Here is the graph of $y = 2^x$ for $0 \leq x \leq 3$ (tangent at $x = 2$ dashed)



(a) Estimate the gradient of the curve at the point where $x = 2$

$$\text{When } x = 2, y = 2^2 = 4$$

Draw a tangent: a straight line that just touches the curve at (2, 4) (the dashed line).

Pick two points on the tangent that are far apart, e.g. (0.6, 0) and (3, 6.8).

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{6.8 - 0}{3 - 0.6} = \frac{6.8}{2.4}$$

$$\approx 2.8$$

Answers close to this are fine, because the tangent is drawn by eye.

Answer: about 2.8 (accept 2.4 to 3.2)

Tip: You must draw a tangent. Dividing the coordinates of the point, $4 \div 2 = 2$, is wrong and gets no marks.

Marks: 1 mark for a tangent drawn at $x = 2$. 1 mark for change in $y \div$ change in x for your tangent. 1 mark for an answer in the accepted range.

- (b) Use 3 strips of equal width to estimate the area under the curve between $x = 0$ and $x = 3$

3 strips from $x = 0$ to $x = 3$, so each strip is 1 wide.

Heights at $x = 0, 1, 2, 3$ are $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$

Each strip is a trapezium: area = $\frac{1}{2}(a + b) \times 1$

$$\frac{1}{2}(1 + 2) + \frac{1}{2}(2 + 4) + \frac{1}{2}(4 + 8)$$

$$= 1.5 + 3 + 6$$

$$= 10.5$$

Answer: 10.5

Marks: 1 mark for the correct heights. 1 mark for adding the trapezium areas. 1 mark for 10.5.

- (c) Is your answer to part (b) an underestimate or an overestimate of the area?
Give a reason for your answer.

The curve bends below the straight tops of the trapezia.

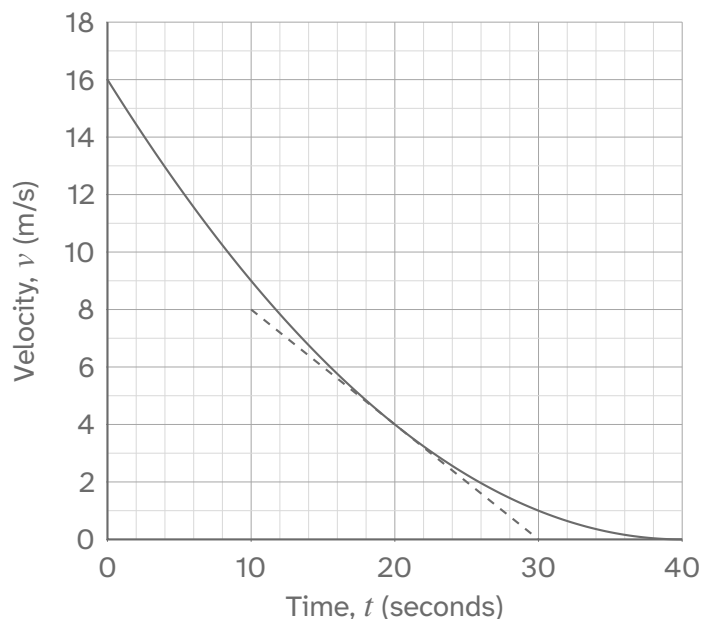
So the trapezia include some extra area above the curve.

Answer: Overestimate: the tops of the trapezia are above the curve

2 (7 marks)

The engine of a speedboat is switched off.

The graph shows the velocity, v m/s, of the speedboat t seconds after the engine is switched off (tangent at $t = 20$ dashed).



- (a) Work out an estimate for the gradient of the graph at $t = 20$
You must show how you get your answer.

At $t = 20$ the velocity is 4 m/s.

Draw a tangent to the curve at (20, 4) (the dashed line).

Two points on the tangent: (10, 8) and (30, 0).

$$\text{Gradient} = \frac{0 - 8}{30 - 10} = \frac{-8}{20}$$

$$= -0.4$$

The gradient is negative because the speedboat is slowing down.

Answer: about -0.4 (accept -0.3 to -0.5)

Tip: The graph slopes down, so the gradient is negative. Dividing the coordinates, $4 \div 20$, is wrong: you must draw a tangent.

Marks: 1 mark for a tangent at $t = 20$. 1 mark for change in $v \div$ change in t for your tangent. 1 mark for an answer in the accepted range.

- (b) Describe fully what your answer to part (a) represents.

The gradient of a velocity-time graph is the acceleration.

It is negative, so the speedboat is decelerating.

The tangent is at $t = 20$, so it is the value at that moment.

Answer: The acceleration of the speedboat 20 seconds after the engine is switched off (a deceleration of about 0.4 m/s^2)

Tip: “Describe fully” means say what it is **and** when. “The speed” or “the distance” is wrong.

- (c) Work out an estimate for the total distance the speedboat travels from when the engine is switched off until the speedboat stops.
Use 4 strips of equal width.

The distance is the area under the velocity-time graph.

The speedboat stops when $v = 0$, at $t = 40$

4 strips from $t = 0$ to $t = 40$, so each strip is 10 wide.

Heights at $t = 0, 10, 20, 30, 40$ are 16, 9, 4, 1, 0

Each strip is a trapezium: area = $\frac{1}{2}(a + b) \times 10$

$$5(16 + 9) + 5(9 + 4) + 5(4 + 1) + 5(1 + 0)$$

$$= 125 + 65 + 25 + 5$$

$$= 220$$

Answer: 220 m

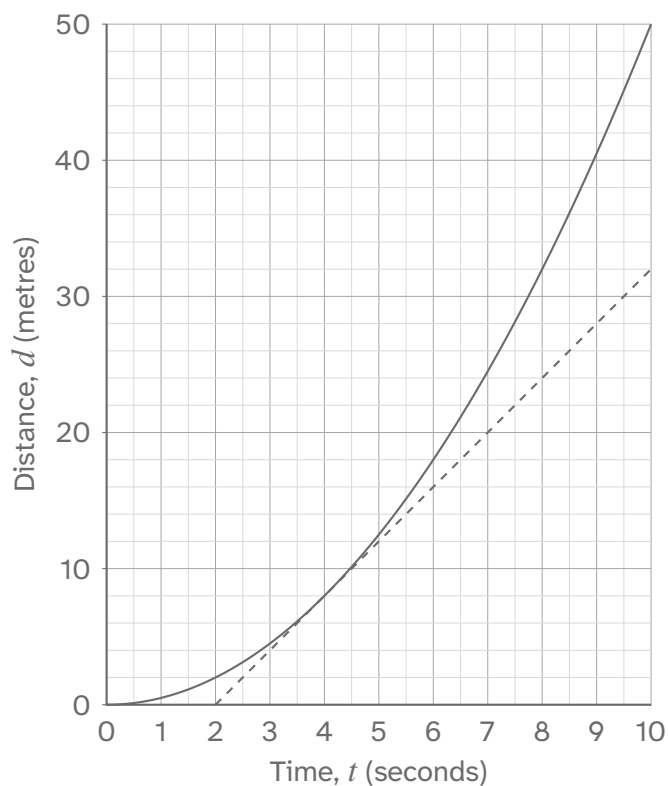
Tip: Check the scale: each small square across is 2 seconds. The last strip is a triangle, so remember to halve it.

Marks: 1 mark for the correct heights. 1 mark for adding the trapezium areas. 1 mark for 220.

3 (4 marks)

A skier starts from rest at the top of a slope.

The distance-time graph shows the distance, d metres, she has travelled down the slope t seconds after she starts (tangent at $t = 4$ dashed).



- (a) By drawing a tangent, work out an estimate for the speed of the skier 4 seconds after she starts.

The speed is the gradient of the distance-time graph.

At $t = 4$ the distance is 8 m.

Draw a tangent at (4, 8) (the dashed line).

Two points on the tangent: (2, 0) and (10, 32).

$$\text{Gradient} = \frac{32 - 0}{10 - 2} = \frac{32}{8}$$

$$= 4$$

Answer: about 4 m/s (accept 3.5 to 4.5)

Tip: Check the scales before you work out the gradient: one small square is 0.5 seconds across and 2 metres up. $8 \div 4 = 2$ is wrong: that is the average speed for the first 4 seconds.

Marks: 1 mark for a tangent at $t = 4$. 1 mark for change in distance $\div$ change in time for your tangent. 1 mark for an answer in the accepted range.

- (b) Describe how the speed of the skier changes during the 10 seconds.
Give a reason for your answer.

The gradient of a distance-time graph is the speed.

The graph gets steeper as t increases.

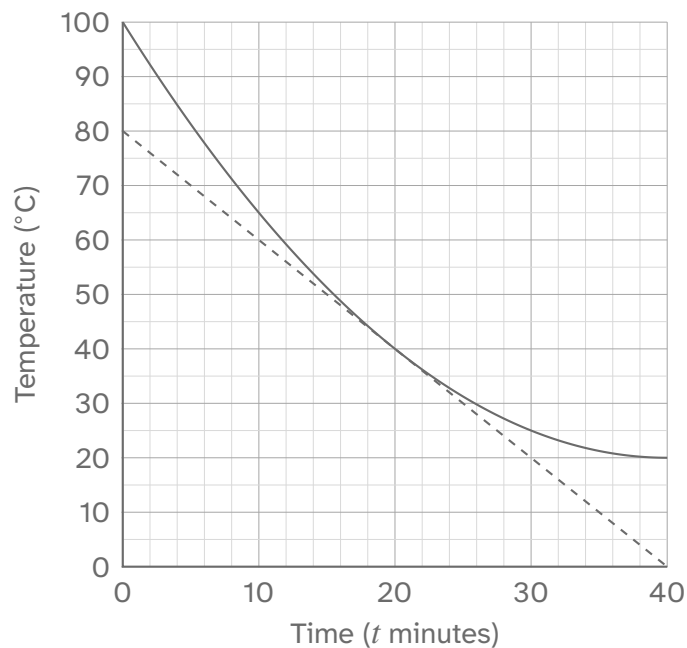
So the skier's speed increases.

Answer: The speed increases, because the graph gets steeper

4 (3 marks)

A cup of hot chocolate is left to cool.

The graph shows the temperature, in °C, of the hot chocolate t minutes after it was poured (tangent at $t = 20$ dashed).



By drawing a tangent, work out an estimate for the rate at which the hot chocolate is cooling 20 minutes after it was poured.

You must show your working.

The rate of cooling is the gradient of the graph.

At $t = 20$ the temperature is 40 °C.

Draw a tangent at (20, 40) (the dashed line).

Two points on the tangent: (0, 80) and (40, 0).

$$\text{Gradient} = \frac{0 - 80}{40 - 0} = \frac{-80}{40}$$

$$= -2$$

The minus sign means the temperature is going down.

So the hot chocolate is cooling at about 2°C per minute.

Answer: about 2°C per minute (accept 1.6 to 2.4)

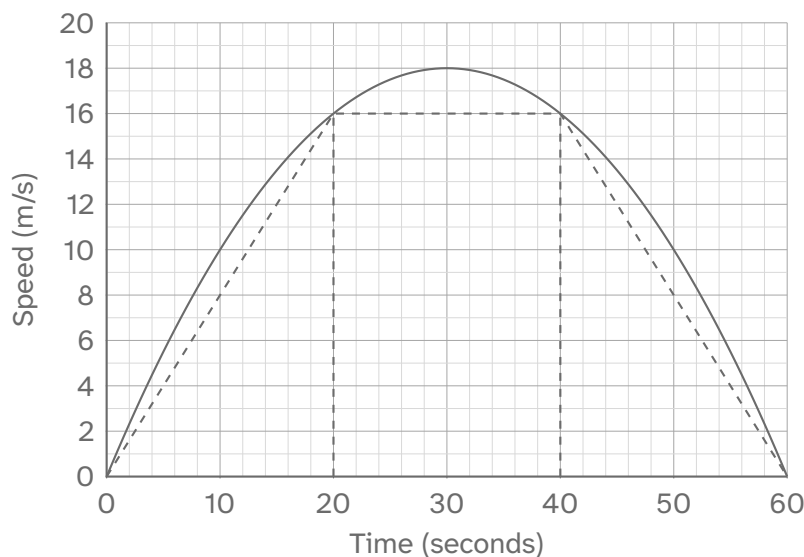
Tip: “Rate” means gradient. Reading the temperature at $t = 20$ and dividing, $40 \div 20$, is wrong. Draw the tangent.

Marks: 1 mark for a tangent at $t = 20$. 1 mark for change in temperature $\div$ change in time for your tangent. 1 mark for an answer in the accepted range.

5 (5 marks)

A tram travels between two stops.

Here is the speed-time graph for the journey (the 3 strips used in part (a) are dashed).



- (a) Work out an estimate for the distance between the two stops.
Use 3 strips of equal width.

The distance is the area under the speed-time graph.

3 strips from 0 to 60 seconds, so each strip is 20 seconds wide.

Speeds at 0, 20, 40, 60 seconds are 0, 16, 16, 0 m/s

Strip 1 (a triangle): $\frac{1}{2} \times 20 \times 16 = 160$

Strip 2 (a rectangle): $20 \times 16 = 320$

Strip 3 (a triangle): $\frac{1}{2} \times 20 \times 16 = 160$

Total: $160 + 320 + 160 = 640$

Answer: 640 m

Marks: 1 mark for the correct speeds. 1 mark for adding the areas of the strips. 1 mark for 640.

- (b) Is your answer to part (a) an underestimate or an overestimate of the actual distance?

Give a reason for your answer.

The curve is above the straight tops of the strips.

So the strips miss some of the area under the curve.

Answer: Underestimate: the curve is above the tops of the strips

Tip: Your reason must say whether the shapes are above or below the curve. "Because it is a curve" or "because I rounded" does not get the mark.

- (c) Kareem says,
"The greatest speed of the tram is 18 m/s and the journey takes 60 seconds.

So the distance between the stops is $18 \times 60 = 1080$ m."

Explain why Kareem is wrong.

Speed $\times$ time only gives the distance when the speed is constant.

The tram only travels at 18 m/s for an instant: most of the time it is slower.

The distance is the area under the graph, not the area of an 18×60 rectangle.

Answer: The speed is not 18 m/s all the time; the distance is the area under the curve, which is less than 18×60

Tip: Distance = speed $\times$ time only works for a constant speed. When the speed changes, find the area under the graph.