

Function Notation

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

$$f(x) = x^2 - 4x$$

(a) Find $f(-3)$

Replace x with -3 , using brackets:

$$f(-3) = (-3)^2 - 4 \times (-3)$$

$$= 9 + 12$$

$$= 21$$

Answer: 21

Tip: Take care with the signs: $(-3)^2 = +9$ and $-4 \times (-3) = +12$.

(b) Solve $f(x) = 12$

$$x^2 - 4x = 12$$

Rearrange so one side is 0: $x^2 - 4x - 12 = 0$

Factorise: $(x - 6)(x + 2) = 0$

So $x - 6 = 0$ or $x + 2 = 0$

Answer: $x = 6$ or $x = -2$

Marks: 1 mark for $x^2 - 4x - 12 = 0$. 1 mark for factorising. 1 mark for both answers.

2 (2 marks)

$$g(x) = \frac{3x}{x-2}$$

(a) Find $g(4)$

$$\begin{aligned} g(4) &= \frac{3 \times 4}{4 - 2} \\ &= \frac{12}{2} = 6 \end{aligned}$$

Answer: 6

(b) Find $g(1)$

$$\begin{aligned} g(1) &= \frac{3 \times 1}{1 - 2} \\ &= \frac{3}{-1} = -3 \end{aligned}$$

Answer: -3

Tip: The bottom is $1 - 2 = -1$. Dividing a positive number by a negative number gives a negative answer.

3 (4 marks)

The function f is such that $f(x) = 3(x - 4)$

(a) Find the value of $f(10)$

$$\begin{aligned} f(10) &= 3(10 - 4) \\ &= 3 \times 6 = 18 \end{aligned}$$

Answer: 18

(b) Solve $f(x) = 15$

$$3(x - 4) = 15$$

Divide both sides by 3: $x - 4 = 5$

Add 4: $x = 9$

Answer: $x = 9$

Marks: 1 mark for $3(x - 4) = 15$. 1 mark for $x = 9$.

- (c) Find an expression for $f(2x)$
Give your answer in its simplest form.

Replace x with $2x$: $f(2x) = 3(2x - 4)$

Expand: $6x - 12$

Answer: $6x - 12$

Tip: $f(2x)$ is not the same as $2f(x)$. Replace each x with $2x$.

4 (1 mark)

$$h(x) = x^2 + 5$$

Dmitri needs to find the value of $h(-3)$

He writes

$$\begin{aligned} h(-3) &= -3^2 + 5 \\ &= -9 + 5 \\ &= -4 \end{aligned}$$

This is not correct.

Explain why.

He should have written $(-3)^2$, with a bracket.

$$(-3)^2 = -3 \times -3 = 9, \text{ not } -9.$$

$$\text{So } h(-3) = 9 + 5 = 14.$$

Dmitri has squared 3 and then made it negative, instead of squaring -3 .

Answer: $(-3)^2 = 9$, not -9 (so $h(-3) = 14$)

Tip: Always put a negative number in brackets when you substitute it: $(-3)^2 = 9$.

5 (3 marks)

The function f is such that $f(x) = ax + b$ where a and b are constants.

$$f(2) = 3 \quad f(-1) = f(4) - 15$$

Find the value of a and the value of b .

$$f(-1) = -a + b \text{ and } f(4) = 4a + b$$

$$\text{So } -a + b = 4a + b - 15$$

$$\text{The } b \text{ terms cancel: } -a = 4a - 15$$

$$15 = 5a, \text{ so } a = 3$$

$$f(2) = 3 \text{ means } 2a + b = 3$$

$$6 + b = 3$$

$$b = -3$$

$$\text{Check: } f(-1) = -6 \text{ and } f(4) - 15 = 9 - 15 = -6 \quad \checkmark$$

Answer: $a = 3, b = -3$

Tip: Write each function value in terms of a and b first. $f(-1)$ is $-a + b$, not $-(a + b)$.

Marks: 1 mark for $-a + b = 4a + b - 15$ (or $2a + b = 3$). 1 mark for $a = 3$. 1 mark for $b = -3$.

6 (3 marks)

$$f(x) = 5x + 1 \text{ and } g(x) = x^2$$

$$\text{Solve } f(x) = g(x) + 7$$

$$\text{Write out the equation: } 5x + 1 = x^2 + 7$$

$$\text{Rearrange so one side is 0: } x^2 - 5x + 6 = 0$$

$$\text{Factorise: } (x - 2)(x - 3) = 0$$

$$\text{So } x - 2 = 0 \text{ or } x - 3 = 0$$

Answer: $x = 2$ or $x = 3$

Marks: 1 mark for $5x + 1 = x^2 + 7$. 1 mark for $x^2 - 5x + 6 = 0$. 1 mark for both answers.

7 (3 marks)

$$h(x) = 2x^2 - x$$

Show that $h(x+2) - h(x) = 8x + 6$

Replace x with $(x+2)$: $h(x+2) = 2(x+2)^2 - (x+2)$

$$(x+2)^2 = x^2 + 4x + 4, \text{ so } 2(x+2)^2 = 2x^2 + 8x + 8$$

$$h(x+2) = 2x^2 + 8x + 8 - x - 2 = 2x^2 + 7x + 6$$

Subtract $h(x)$, using a bracket:

$$2x^2 + 7x + 6 - (2x^2 - x)$$

$$= 2x^2 + 7x + 6 - 2x^2 + x$$

$$= 8x + 6$$

Answer: Shown

Tip: Square the bracket first, then multiply by 2: $2(x+2)^2$ is not $(2x+4)^2$.

Marks: 1 mark for $2(x+2)^2 - (x+2)$. 1 mark for $2x^2 + 7x + 6$. 1 mark for a fully correct method.

8 (3 marks)

$$f(x) = x^2 + 2x \text{ and } g(x) = 4x + 3$$

$$\text{Solve } f(x) = 2g(x) - 1$$

Give your answers correct to 3 significant figures.

$$2g(x) - 1 = 2(4x + 3) - 1 = 8x + 5$$

$$\text{So } x^2 + 2x = 8x + 5$$

$$\text{Rearrange: } x^2 - 6x - 5 = 0$$

Use the formula with $a = 1$, $b = -6$, $c = -5$:

$$x = \frac{6 \pm \sqrt{36 + 20}}{2} = \frac{6 \pm \sqrt{56}}{2}$$

$$x = 6.7416 \dots \text{ or } x = -0.7416 \dots$$

Answer: $x = 6.74$ or $x = -0.742$

Tip: For 3 significant figures, $-0.7416 \dots$ becomes -0.742 . The zero at the start does not count as a significant figure.

Marks: 1 mark for $x^2 + 2x = 8x + 5$. 1 mark for $x^2 - 6x - 5 = 0$ and using the formula. 1 mark for both answers to 3 significant figures.
