

Factorising Harder Quadratics

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Fill in the boxes to complete each factorisation.

(a) $2x^2 + 7x + 3 = (2x + \boxed{})(x + \boxed{})$

The two numbers in the boxes must multiply to give 3.

So they are 1 and 3, in some order.

Try $(2x + 1)(x + 3)$:

$$2x^2 + 6x + x + 3 = 2x^2 + 7x + 3 \quad \checkmark$$

Answer: $(2x + 1)(x + 3)$

(b) $3x^2 - 10x - 8 = (3x + \boxed{})(x - \boxed{})$

The two numbers in the boxes must multiply to give 8.

Try the pairs 1, 8 and 2, 4 in both orders.

Try $(3x + 2)(x - 4)$:

$$3x^2 - 12x + 2x - 8 = 3x^2 - 10x - 8 \quad \checkmark$$

Answer: $(3x + 2)(x - 4)$

2 (4 marks)

(a) Factorise $5x^2 + 14x - 3$

Multiply the first and last numbers: $5 \times (-3) = -15$

Find two numbers that multiply to -15 and add to 14 : 15 and -1

Split the middle term: $5x^2 + 15x - x - 3$

Factorise in pairs: $5x(x + 3) - 1(x + 3)$

Both parts have $(x + 3)$, so take it out.

Answer: $(5x - 1)(x + 3)$

Marks: 1 mark for $(5x \pm 1)(x \pm 3)$. 1 mark for the correct answer.

(b) Factorise $4x^2 - 4x - 15$

Multiply the first and last numbers: $4 \times (-15) = -60$

Find two numbers that multiply to -60 and add to -4 : -10 and 6

Split the middle term: $4x^2 - 10x + 6x - 15$

Factorise in pairs: $2x(2x - 5) + 3(2x - 5)$

Take out $(2x - 5)$.

Check: $(2x - 5)(2x + 3) = 4x^2 + 6x - 10x - 15 = 4x^2 - 4x - 15$ ✓

Answer: $(2x - 5)(2x + 3)$

Tip: Sign mistakes are very common, e.g. $(2x + 5)(2x - 3)$. Expand your brackets to check you get back to $-4x$.

Marks: 1 mark for brackets that give two of the three terms. 1 mark for the correct answer.

3 (2 marks)

Factorise fully $50 - 2x^2$

First take out the common factor 2:

$$50 - 2x^2 = 2(25 - x^2)$$

$25 - x^2 = 5^2 - x^2$ is a difference of two squares.

$$\text{So } 25 - x^2 = (5 + x)(5 - x)$$

Answer: $2(5 + x)(5 - x)$

Tip: Always look for a common factor first. The 2 must stay in your final answer.

Marks: 1 mark for $2(25 - x^2)$ or $(10 + 2x)(5 - x)$. 1 mark for the correct answer.

4 (3 marks)

Solve $6x^2 - 7x - 3 = 0$

Factorise the left side.

$6 \times (-3) = -18$, and -9 and 2 multiply to -18 and add to -7 .

$$6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3)$$

$$\text{So } (3x + 1)(2x - 3) = 0$$

If two things multiply to make 0, one of them must be 0.

$$3x + 1 = 0, \text{ so } x = -\frac{1}{3}$$

$$2x - 3 = 0, \text{ so } x = \frac{3}{2}$$

Answer: $x = -\frac{1}{3}, x = \frac{3}{2}$

Marks: 1 mark for $(3x \pm 1)(2x \pm 3)$. 1 mark for $(3x + 1)(2x - 3)$. 1 mark for both solutions.

5 (2 marks)

Factorise fully $x^4 - 81$

$$x^4 = (x^2)^2 \text{ and } 81 = 9^2$$

So $x^4 - 81 = (x^2)^2 - 9^2$, a difference of two squares.

$$x^4 - 81 = (x^2 - 9)(x^2 + 9)$$

$x^2 - 9 = x^2 - 3^2$ is also a difference of two squares:

$$x^2 - 9 = (x - 3)(x + 3)$$

$x^2 + 9$ does not factorise.

Answer: $(x - 3)(x + 3)(x^2 + 9)$

Tip: “Fully” is a clue that there is more than one step. After $(x^2 - 9)(x^2 + 9)$, check whether a bracket factorises again.

Marks: 1 mark for $(x^2 - 9)(x^2 + 9)$. 1 mark for the correct answer.

6 (3 marks)

$(2x + 1)$ is a factor of $2x^2 + kx - 5$

Find the value of k and factorise the expression.

The other factor must start with x , because $2x \times x = 2x^2$.

So $2x^2 + kx - 5 = (2x + 1)(x + c)$ for some number c .

Compare the number terms: $1 \times c = -5$, so $c = -5$

Expand: $(2x + 1)(x - 5) = 2x^2 - 10x + x - 5 = 2x^2 - 9x - 5$

So $k = -9$

Answer: $k = -9$ and $2x^2 - 9x - 5 = (2x + 1)(x - 5)$

Marks: 1 mark for the other factor $(x - 5)$ (or $c = -5$). 1 mark for $k = -9$. 1 mark for $2x^2 - 9x - 5 = (2x + 1)(x - 5)$.

7 (2 marks)

Factorise $4k^2 + 11k - 3$

Multiply the first and last numbers: $4 \times (-3) = -12$

Find two numbers that multiply to -12 and add to 11 : 12 and -1

Split the middle term: $4k^2 + 12k - k - 3$

Factorise in pairs: $4k(k + 3) - 1(k + 3)$

Take out $(k + 3)$.

Answer: $(4k - 1)(k + 3)$

Tip: $k(4k + 11) - 3$ is not factorised. Your answer must be two brackets multiplied together.

Marks: 1 mark for $(4k \pm 1)(k \pm 3)$. 1 mark for the correct answer.

8 (6 marks)

- (a) Show, by factorising, that $1002^2 - 998^2 = 8000$

This is a difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

Use $a = 1002$ and $b = 998$.

$$a + b = 1002 + 998 = 2000$$

$$a - b = 1002 - 998 = 4$$

$$1002^2 - 998^2 = 2000 \times 4 = 8000$$

Answer: $(1002 + 998)(1002 - 998) = 2000 \times 4 = 8000$

Tip: The question says “by factorising”, so working out 1002^2 and 998^2 on a calculator gets no marks. Show the brackets.

Marks: 1 mark for $(1002 + 998)(1002 - 998)$. 1 mark for $2000 \times 4 = 8000$.

- (b) Factorise $(x + 4)^2 - 9$

$9 = 3^2$, so this is a difference of two squares with $a = x + 4$ and $b = 3$.

$$(x + 4 + 3)(x + 4 - 3)$$

$$= (x + 7)(x + 1)$$

Answer: $(x + 7)(x + 1)$

- (c) Factorise fully $(3x + 2)^2 - (x - 4)^2$

Use $a^2 - b^2 = (a + b)(a - b)$ with $a = 3x + 2$ and $b = x - 4$.

$$a + b = 3x + 2 + x - 4 = 4x - 2$$

$$a - b = 3x + 2 - (x - 4) = 3x + 2 - x + 4 = 2x + 6$$

So the expression is $(4x - 2)(2x + 6)$

Each bracket has a common factor 2: $4x - 2 = 2(2x - 1)$ and $2x + 6 = 2(x + 3)$

$$= 2(2x - 1) \times 2(x + 3) = 4(2x - 1)(x + 3)$$

(Or expand first:

$$9x^2 + 12x + 4 - (x^2 - 8x + 16) = 8x^2 + 20x - 12 = 4(2x^2 + 5x - 3) = 4(2x - 1)(x + 3).)$$

Answer: $4(2x - 1)(x + 3)$

Tip: Put brackets round b when you subtract it: $3x + 2 - (x - 4) = 2x + 6$, not $2x - 2$. Then check each bracket for a common factor, because “fully” means the 4 must come out.

Marks: 1 mark for $(3x + 2 + x - 4)(3x + 2 - (x - 4))$ or $8x^2 + 20x - 12$. 1 mark for $(4x - 2)(2x + 6)$ or $4(2x^2 + 5x - 3)$. 1 mark for $4(2x - 1)(x + 3)$.
