

Expanding Double and Triple Brackets

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Expand and simplify $(3x - 2)(2x + 5)$

Multiply each term in the first bracket by each term in the second:

$$3x \times 2x = 6x^2$$

$$3x \times 5 = 15x$$

$$-2 \times 2x = -4x$$

$$-2 \times 5 = -10$$

So we get $6x^2 + 15x - 4x - 10$

Collect the x terms: $15x - 4x = 11x$

Answer: $6x^2 + 11x - 10$

Marks: 1 mark for four terms with at least three correct. 1 mark for the correct answer.

2 (2 marks)

Expand and simplify $(2x - 3)^2$

$(2x - 3)^2$ means $(2x - 3)(2x - 3)$.

$$2x \times 2x = 4x^2$$

$$2x \times (-3) = -6x$$

$$-3 \times 2x = -6x$$

$$-3 \times (-3) = +9$$

So we get $4x^2 - 6x - 6x + 9$

Answer: $4x^2 - 12x + 9$

Marks: 1 mark for four terms with at least three correct. 1 mark for the correct answer.

3 (2 marks)

Expand and simplify $(2x + 3y)^2$

$(2x + 3y)^2$ means $(2x + 3y)(2x + 3y)$.

$$2x \times 2x = 4x^2$$

$$2x \times 3y = 6xy$$

$$3y \times 2x = 6xy$$

$$3y \times 3y = 9y^2$$

So we get $4x^2 + 6xy + 6xy + 9y^2$

Answer: $4x^2 + 12xy + 9y^2$

Tip: A very common wrong answer is $4x^2 + 9y^2$. Squaring a bracket always gives a middle term, here $12xy$.

Marks: 1 mark for four terms with at least three correct. 1 mark for the correct answer.

4 (3 marks)

Expand and simplify $(x + 6)(x - 3)(x + 2)$

Expand the first two brackets:

$$(x + 6)(x - 3) = x^2 - 3x + 6x - 18 = x^2 + 3x - 18$$

Now multiply by $(x + 2)$:

$$x^2 \times x = x^3 \quad x^2 \times 2 = 2x^2$$

$$3x \times x = 3x^2 \quad 3x \times 2 = 6x$$

$$-18 \times x = -18x \quad -18 \times 2 = -36$$

So we get $x^3 + 2x^2 + 3x^2 + 6x - 18x - 36$

Collect like terms.

Answer: $x^3 + 5x^2 - 12x - 36$

Tip: Do not try to multiply all three brackets at once. Expand two brackets, simplify, then multiply by the third. You should get six terms before you simplify.

Marks: 1 mark for two brackets expanded (at least three terms correct). 1 mark for six terms (at least four correct). 1 mark for the correct answer.

5 (3 marks)

Expand and simplify $(x - 1)^3$

$(x - 1)^3$ means $(x - 1)(x - 1)(x - 1)$.

First: $(x - 1)^2 = (x - 1)(x - 1) = x^2 - 2x + 1$

Now multiply by $(x - 1)$:

$$x^2 \times x = x^3 \quad x^2 \times (-1) = -x^2$$

$$-2x \times x = -2x^2 \quad -2x \times (-1) = 2x$$

$$1 \times x = x \quad 1 \times (-1) = -1$$

So we get $x^3 - x^2 - 2x^2 + 2x + x - 1$

Collect like terms.

Answer: $x^3 - 3x^2 + 3x - 1$

Marks: 1 mark for $x^2 - 2x + 1$. 1 mark for six terms. 1 mark for the correct answer.

6 (3 marks)

Show that $(x + 3)(2x - 1)(x - 4)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a , b , c and d are integers.

You can multiply the brackets in any order.

Expand $(x + 3)(x - 4)$ first:

$$(x + 3)(x - 4) = x^2 - 4x + 3x - 12 = x^2 - x - 12$$

Now multiply by $(2x - 1)$:

$$(x^2 - x - 12)(2x - 1) = 2x^3 - x^2 - 2x^2 + x - 24x + 12$$

Collect like terms:

$$= 2x^3 - 3x^2 - 23x + 12$$

This is in the form $ax^3 + bx^2 + cx + d$ with $a = 2$, $b = -3$, $c = -23$ and $d = 12$, which are all integers.

Answer: $2x^3 - 3x^2 - 23x + 12$

Marks: 1 mark for two brackets expanded. 1 mark for six terms. 1 mark for the correct final expression.

7 (3 marks)

Expand and simplify $(x - 4)(2x + 1)(3x + 2)$

Expand $(2x + 1)(3x + 2)$ first:

$$(2x + 1)(3x + 2) = 6x^2 + 4x + 3x + 2 = 6x^2 + 7x + 2$$

Now multiply by $(x - 4)$:

$$6x^2 \times x = 6x^3 \quad 6x^2 \times (-4) = -24x^2$$

$$7x \times x = 7x^2 \quad 7x \times (-4) = -28x$$

$$2 \times x = 2x \quad 2 \times (-4) = -8$$

$$\text{So we get } 6x^3 - 24x^2 + 7x^2 - 28x + 2x - 8$$

Collect like terms:

$$-24x^2 + 7x^2 = -17x^2$$

$$-28x + 2x = -26x$$

Answer: $6x^3 - 17x^2 - 26x - 8$

Tip: Most lost marks here come from signs when collecting terms. Check $-24 + 7 = -17$ and $-28 + 2 = -26$ carefully.

Marks: 1 mark for two brackets expanded. 1 mark for six terms. 1 mark for the correct answer.

8 (3 marks)

n is an integer.

Prove that $(2n + 5)(n - 1) - (n + 2)(2n - 3)$ is always an odd number.

Expand each pair of brackets:

$$(2n + 5)(n - 1) = 2n^2 - 2n + 5n - 5 = 2n^2 + 3n - 5$$

$$(n + 2)(2n - 3) = 2n^2 - 3n + 4n - 6 = 2n^2 + n - 6$$

Subtract, using a bracket:

$$2n^2 + 3n - 5 - (2n^2 + n - 6)$$

$$= 2n^2 + 3n - 5 - 2n^2 - n + 6$$

$$= 2n + 1$$

$2n$ is $2 \times$ an integer, so $2n$ is always even.

An even number $+1$ is always odd, so $2n + 1$ is always odd.

Answer: $2n + 1$, which is always odd

Tip: When you subtract $(n + 2)(2n - 3)$, put its expansion in a bracket first. Every sign changes:
 $-(2n^2 + n - 6) = -2n^2 - n + 6$.

Marks: 1 mark for expanding one pair of brackets correctly. 1 mark for $2n + 1$. 1 mark for a conclusion that $2n + 1$ is odd because $2n$ is even.

9 (3 marks)

$$(x + 4)(x - 1)(x + k) \equiv x^3 + ax^2 + 5x + b$$

Find the values of k , a and b .

Expand the first two brackets:

$$(x + 4)(x - 1) = x^2 - x + 4x - 4 = x^2 + 3x - 4$$

Multiply by $(x + k)$:

$$(x^2 + 3x - 4)(x + k) = x^3 + kx^2 + 3x^2 + 3kx - 4x - 4k$$

$$= x^3 + (k + 3)x^2 + (3k - 4)x - 4k$$

Compare the x terms: $3k - 4 = 5$

So $3k = 9$ and $k = 3$

Compare the x^2 terms: $a = k + 3 = 3 + 3 = 6$

Compare the number terms: $b = -4k = -4 \times 3 = -12$

Answer: $k = 3$, $a = 6$, $b = -12$

Tip: Here the number term is unknown, so use the x term instead: collect every term in x to get $(3k - 4)x$ and set it equal to $5x$.

Marks: 1 mark for expanding to get $(3k - 4)x$ (or all the terms in k). 1 mark for $k = 3$. 1 mark for $a = 6$ and $b = -12$.