

Exact Trig Values - Higher

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

(a) Write down the exact value of $\sin 45^\circ$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\text{Rationalise: } \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$$

Answer: $\frac{\sqrt{2}}{2}$ (or $\frac{1}{\sqrt{2}}$)

(b) Write down the exact value of $\tan 60^\circ$

$$\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Answer: $\sqrt{3}$

Tip: $\tan 60^\circ = \sqrt{3}$ and $\tan 30^\circ = \frac{1}{\sqrt{3}}$. A common mistake is to swap them.

(c) x is an acute angle.

$$\cos x = \frac{\sqrt{3}}{2}$$

Write down the value of x .

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

So $x = 30^\circ$

Answer: $x = 30^\circ$

Tip: $\cos 60^\circ = \frac{1}{2}$, not $\frac{\sqrt{3}}{2}$. The cosine values go *down* as the angle goes up:
 $\cos 30^\circ > \cos 45^\circ > \cos 60^\circ$.

2 (2 marks)

Find the exact value of $\tan 60^\circ \times \cos 45^\circ$

Give your answer in the form $\frac{\sqrt{a}}{b}$ where a and b are integers.

$$\tan 60^\circ = \sqrt{3} \text{ and } \cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\sqrt{3} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{3} \times \sqrt{2}}{2}$$

$$\sqrt{3} \times \sqrt{2} = \sqrt{6}, \text{ so the answer is } \frac{\sqrt{6}}{2}$$

Answer: $\frac{\sqrt{6}}{2}$

Tip: $\sqrt{3} \times \sqrt{2} = \sqrt{6}$, not $\sqrt{5}$. Multiply the numbers under the roots.

Marks: 1 mark for both exact values. 1 mark for $\frac{\sqrt{6}}{2}$.

3 (2 marks)

Write $12 \div \sin 60^\circ$ in the form $a\sqrt{3}$ where a is an integer.

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$12 \div \frac{\sqrt{3}}{2} = 12 \times \frac{2}{\sqrt{3}} = \frac{24}{\sqrt{3}}$$

Rationalise: multiply the top and bottom by $\sqrt{3}$

$$\frac{24 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}$$

Answer: $8\sqrt{3}$

Tip: Dividing by $\frac{\sqrt{3}}{2}$ is the same as multiplying by $\frac{2}{\sqrt{3}}$. Do not just divide 12 by 2.

Marks: 1 mark for $\frac{24}{\sqrt{3}}$ (or $12 \div \frac{\sqrt{3}}{2}$). 1 mark for $8\sqrt{3}$.

4 (2 marks)

Write $(2 \sin 60^\circ - 3)^2$ in the form $a + b\sqrt{3}$ where a and b are integers.

$$2 \sin 60^\circ = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$\text{So } (2 \sin 60^\circ - 3)^2 = (\sqrt{3} - 3)^2 = (\sqrt{3} - 3)(\sqrt{3} - 3)$$

$$\sqrt{3} \times \sqrt{3} = 3$$

$$\sqrt{3} \times (-3) = -3\sqrt{3}, \text{ twice: } -6\sqrt{3}$$

$$(-3) \times (-3) = 9$$

$$3 - 6\sqrt{3} + 9 = 12 - 6\sqrt{3}$$

Answer: $12 - 6\sqrt{3}$

Tip: $2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$. A common mistake is to write $\frac{2\sqrt{3}}{4}$. Also, $(\sqrt{3} - 3)^2$ is not $3 + 9$: you need the middle terms.

Marks: 1 mark for $(\sqrt{3} - 3)^2$ expanded with at least three of the four terms correct. 1 mark for $12 - 6\sqrt{3}$.

5 (3 marks)

ABC is a right-angled triangle.

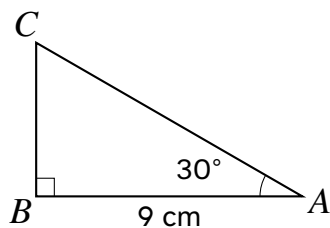


Diagram **NOT** accurately drawn

Work out the length of AC .

Give your answer in the form $k\sqrt{3}$ where k is an integer.

From the 30° angle at A : $AB = 9$ cm is the adjacent and AC is the hypotenuse.

$$\text{CAH: } \cos 30^\circ = \frac{9}{AC}$$

$$AC = \frac{9}{\cos 30^\circ} = 9 \div \frac{\sqrt{3}}{2} = \frac{18}{\sqrt{3}}$$

$$\text{Rationalise: } \frac{18 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{18\sqrt{3}}{3}$$

$$= 6\sqrt{3}$$

Answer: $6\sqrt{3}$ cm

Tip: AC is on the bottom of the fraction, so $AC = \frac{9}{\cos 30^\circ}$, not $9 \times \cos 30^\circ$. The hypotenuse must be longer than 9 cm.

Marks: 1 mark for $\cos 30^\circ = \frac{9}{AC}$ (or $AC = \frac{9}{\cos 30^\circ}$). 1 mark for $\frac{18}{\sqrt{3}}$. 1 mark for $6\sqrt{3}$.

6 (3 marks)

The diagram shows two right-angled triangles, ABC and ACD .

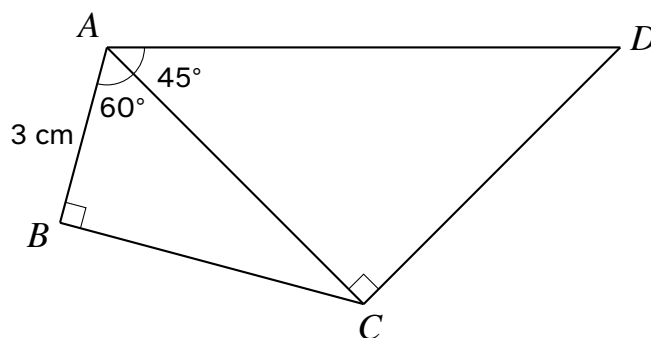


Diagram **NOT** accurately drawn

Work out the length of AD .

Give your answer as a surd in its simplest form.

Triangle ABC : $AB = 3$ cm is adjacent to the 60° angle and AC is the hypotenuse.

$$\cos 60^\circ = \frac{3}{AC}, \text{ so } AC = \frac{3}{\frac{1}{2}} = 6 \text{ cm}$$

Triangle ACD : $AC = 6$ cm is adjacent to the 45° angle and AD is the hypotenuse.

$$\cos 45^\circ = \frac{6}{AD}, \text{ so } AD = 6 \div \frac{\sqrt{2}}{2} = \frac{12}{\sqrt{2}}$$

$$\text{Rationalise: } \frac{12\sqrt{2}}{2} = 6\sqrt{2}$$

Answer: $6\sqrt{2}$ cm

Tip: Triangle ACD has a 45° angle, so it is isosceles: $CD = AC = 6$ cm. Pythagoras then gives $AD = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$.

Marks: 1 mark for $AC = 6$. 1 mark for $AD = \frac{6}{\cos 45^\circ}$ (or Pythagoras with $CD = 6$). 1 mark for $6\sqrt{2}$.

7 (3 marks)

Show that $\frac{1 + \tan 60^\circ}{\tan 60^\circ - 1} = 2 + \sqrt{3}$

$$\tan 60^\circ = \sqrt{3}, \text{ so the fraction is } \frac{1 + \sqrt{3}}{\sqrt{3} - 1}$$

Rationalise: multiply the top and bottom by $\sqrt{3} + 1$

Bottom: $(\sqrt{3} - 1)(\sqrt{3} + 1) = 3 + \sqrt{3} - \sqrt{3} - 1 = 2$

Top: $(1 + \sqrt{3})(\sqrt{3} + 1) = \sqrt{3} + 1 + 3 + \sqrt{3} = 4 + 2\sqrt{3}$

$$\frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3} \quad \checkmark$$

Answer: $\frac{1 + \sqrt{3}}{\sqrt{3} - 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$

Tip: To rationalise $\frac{1}{\sqrt{3}-1}$, multiply by $\sqrt{3} + 1$ (change the sign). Multiplying by $\sqrt{3}$ does not remove the surd from the bottom.

Marks: 1 mark for replacing $\tan 60^\circ$ with $\sqrt{3}$. 1 mark for multiplying the top and bottom by $\sqrt{3} + 1$. 1 mark for $\frac{4+2\sqrt{3}}{2}$ simplified to $2 + \sqrt{3}$.

8 (4 marks)

ABC is a right-angled triangle.

D is the point on AB such that $AD = 4$ cm and $DB = 2$ cm.

Angle $BAC = 60^\circ$

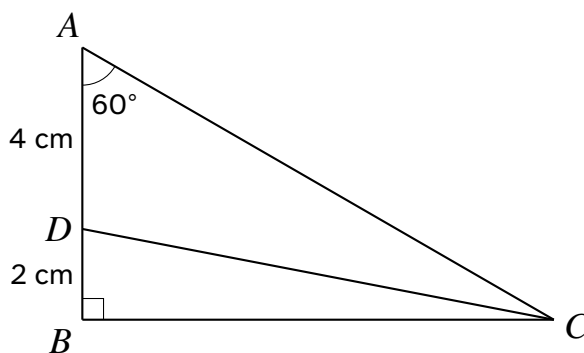


Diagram **NOT** accurately drawn

Work out the exact length of CD .

Give your answer as a surd in its simplest form.

You must show all your working.

$$AB = 4 + 2 = 6 \text{ cm}$$

In triangle ABC , BC is opposite the 60° angle and AB is adjacent.

$$\text{TOA: } \tan 60^\circ = \frac{BC}{6}$$

$$BC = 6 \tan 60^\circ = 6\sqrt{3} \text{ cm}$$

Triangle DBC has a right angle at B , with $DB = 2$ cm.

Pythagoras: $CD^2 = 2^2 + (6\sqrt{3})^2$

$$(6\sqrt{3})^2 = 36 \times 3 = 108, \text{ so } CD^2 = 4 + 108 = 112$$

$$CD = \sqrt{112} = \sqrt{16} \times \sqrt{7} = 4\sqrt{7}$$

Answer: $4\sqrt{7}$ cm

Tip: Use $AB = 6$ cm (not 4 cm) with the 60° angle. And $(6\sqrt{3})^2 = 108$, not $6 \times 3 = 18$: square the 6 as well.

Marks: 1 mark for $6 \tan 60^\circ$. 1 mark for $BC = 6\sqrt{3}$. 1 mark for $2^2 + (6\sqrt{3})^2$ (or 112). 1 mark for $4\sqrt{7}$.

9 (3 marks)

Here is triangle PQR .

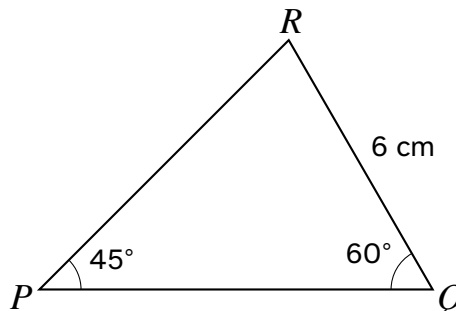


Diagram **NOT** accurately drawn

Work out the length of PR .

Give your answer as a surd in its simplest form.

The triangle is not right-angled, so use the sine rule.

PR is opposite the 60° angle. $QR = 6$ cm is opposite the 45° angle.

$$\frac{PR}{\sin 60^\circ} = \frac{6}{\sin 45^\circ}$$

$$PR = \frac{6 \sin 60^\circ}{\sin 45^\circ} = \frac{6 \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} = \frac{3\sqrt{3}}{\frac{\sqrt{2}}{2}}$$

$$= 3\sqrt{3} \times \frac{2}{\sqrt{2}} = \frac{6\sqrt{3}}{\sqrt{2}}$$

$$\text{Rationalise: } \frac{6\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{6\sqrt{6}}{2} = 3\sqrt{6}$$

Answer: $3\sqrt{6}$ cm

Tip: Do not use SOHCAHTOA here: there is no right angle. Pair each side with the angle *opposite* it.

Marks: 1 mark for $\frac{PR}{\sin 60^\circ} = \frac{6}{\sin 45^\circ}$. 1 mark for substituting both exact values. 1 mark for $3\sqrt{6}$.

10 (4 marks)

The diagram shows a regular hexagon $ABCDEF$.
Each side of the hexagon is 6 cm long.

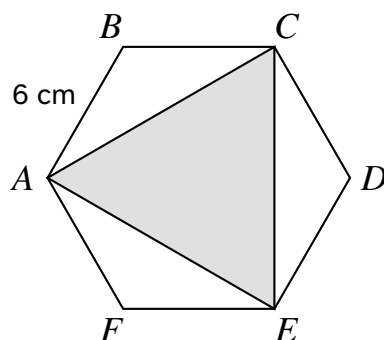


Diagram **NOT** accurately drawn

Work out the exact area of the shaded triangle ACE .

Give your answer in the form $k\sqrt{3}$

You must show all your working.

Each interior angle of a regular hexagon is 120° , so angle $ABC = 120^\circ$.

Triangle ABC is isosceles ($AB = BC = 6$). Split it in half to make two right-angled triangles with angle 30° at A .

$$\text{Half of } AC = 6 \cos 30^\circ = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}, \text{ so } AC = 6\sqrt{3} \text{ cm}$$

Triangle ACE is equilateral (by symmetry, $AC = CE = EA$), so every angle is 60° .

$$\text{Area} = \frac{1}{2} \times AC \times AE \times \sin 60^\circ = \frac{1}{2} \times 6\sqrt{3} \times 6\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$6\sqrt{3} \times 6\sqrt{3} = 36 \times 3 = 108$$

$$\text{Area} = \frac{1}{2} \times 108 \times \frac{\sqrt{3}}{2} = 27\sqrt{3}$$

Answer: $27\sqrt{3} \text{ cm}^2$

Tip: AC is a diagonal, not a side, so it is not 6 cm. Another way: the hexagon is six equilateral triangles, each $\frac{1}{2} \times 6 \times 6 \times \sin 60^\circ = 9\sqrt{3}$, so $54\sqrt{3}$ in total. Triangle ABC is $\frac{1}{2} \times 6 \times 6 \times \sin 120^\circ = 9\sqrt{3}$ (as $\sin 120^\circ = \sin 60^\circ$). Then $54\sqrt{3} - 3 \times 9\sqrt{3} = 27\sqrt{3}$.

Marks: 1 mark for using angle $ABC = 120^\circ$ (or angle $BAC = 30^\circ$). 1 mark for $AC = 6\sqrt{3}$. 1 mark for $\frac{1}{2} \times 6\sqrt{3} \times 6\sqrt{3} \times \sin 60^\circ$. 1 mark for $27\sqrt{3}$.
