

Equations with Algebraic Fractions

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Solve $\frac{6}{x-2} = 3$

Multiply both sides by $(x - 2)$:

$$6 = 3(x - 2)$$

$$6 = 3x - 6$$

Add 6 to both sides: $12 = 3x$

Divide both sides by 3: $x = 4$

Answer: $x = 4$

Marks: 1 mark for $6 = 3(x - 2)$. 1 mark for $x = 4$.

2 (3 marks)

Solve $\frac{x+3}{2} = \frac{6}{x-1}$

Multiply both sides by $2(x - 1)$. This is the same as cross-multiplying:

$$(x + 3)(x - 1) = 2 \times 6$$

Expand: $x^2 + 2x - 3 = 12$

Rearrange so one side is 0: $x^2 + 2x - 15 = 0$

Factorise: $(x + 5)(x - 3) = 0$

So $x + 5 = 0$ or $x - 3 = 0$

Answer: $x = -5$ or $x = 3$

Marks: 1 mark for cross-multiplying correctly. 1 mark for $x^2 + 2x - 15 = 0$. 1 mark for both answers.

3 (4 marks)

Solve $\frac{3}{x+2} + \frac{2}{x} = 1$

Multiply every term by $x(x+2)$ to clear the fractions:

$$3x + 2(x+2) = x(x+2)$$

Expand: $3x + 2x + 4 = x^2 + 2x$

Simplify: $5x + 4 = x^2 + 2x$

Rearrange: $x^2 - 3x - 4 = 0$

Factorise: $(x-4)(x+1) = 0$

So $x - 4 = 0$ or $x + 1 = 0$

Answer: $x = 4$ or $x = -1$

Tip: Each numerator gets multiplied by the **other** denominator: $\frac{3}{x+2} \times x(x+2) = 3x$. Multiply the 1 on the right as well.

Marks: 1 mark for multiplying by $x(x+2)$. 1 mark for $x^2 - 3x - 4 = 0$. 1 mark for factorising. 1 mark for both answers.

4 (5 marks)

Solve $\frac{2}{x-3} + \frac{4}{x+1} = 1$

Multiply every term by $(x-3)(x+1)$:

$$2(x+1) + 4(x-3) = (x-3)(x+1)$$

Expand the left: $2x + 2 + 4x - 12 = 6x - 10$

Expand the right: $x^2 - 2x - 3$

So $6x - 10 = x^2 - 2x - 3$

Rearrange: $x^2 - 8x + 7 = 0$

Factorise: $(x - 1)(x - 7) = 0$

So $x - 1 = 0$ or $x - 7 = 0$

Answer: $x = 1$ or $x = 7$

Tip: Move everything to the side with the x^2 term, so the x^2 stays positive. Take care with the signs as you do this.

Marks: 1 mark for multiplying by $(x - 3)(x + 1)$. 1 mark for expanding correctly. 1 mark for $x^2 - 8x + 7 = 0$. 1 mark for factorising. 1 mark for both answers.

5 (5 marks)

Solve $\frac{3}{x-1} - \frac{1}{x+2} = \frac{1}{2}$

Give your answers correct to 2 decimal places.

Multiply every term by $2(x - 1)(x + 2)$:

$$\frac{3}{x-1} \times 2(x-1)(x+2) = 6(x+2)$$

$$\frac{1}{x+2} \times 2(x-1)(x+2) = 2(x-1)$$

$$\frac{1}{2} \times 2(x-1)(x+2) = (x-1)(x+2)$$

$$\text{So } 6(x+2) - 2(x-1) = (x-1)(x+2)$$

$$\text{Left: } 6x + 12 - 2x + 2 = 4x + 14$$

$$\text{Right: } x^2 + 2x - x - 2 = x^2 + x - 2$$

$$\text{So } 4x + 14 = x^2 + x - 2$$

$$\text{Rearrange: } x^2 - 3x - 16 = 0$$

This does not factorise, so use the formula with $a = 1$, $b = -3$, $c = -16$:

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-16)}}{2 \times 1} = \frac{3 \pm \sqrt{73}}{2}$$

$$x = 5.772... \text{ or } x = -2.772...$$

Answer: $x = 5.77$ or $x = -2.77$

Tip: The minus sign belongs to the whole of $2(x - 1)$: $-2(x - 1) = -2x + 2$, not $-2x - 2$.

Marks: 1 mark for multiplying by $2(x - 1)(x + 2)$ (or a common denominator). 1 mark for $4x + 14$ on the left. 1 mark for $x^2 - 3x - 16 = 0$. 1 mark for substituting into the formula correctly. 1 mark for both answers to 2 decimal places.

6 (4 marks)

Solve $\frac{x}{x-1} + \frac{2}{x+3} = 3$

Give your answers in the form $\frac{a \pm \sqrt{b}}{c}$ where a , b and c are integers.

Multiply every term by $(x - 1)(x + 3)$:

$$x(x + 3) + 2(x - 1) = 3(x - 1)(x + 3)$$

$$\text{Left: } x^2 + 3x + 2x - 2 = x^2 + 5x - 2$$

$$\text{Right: } 3(x^2 + 2x - 3) = 3x^2 + 6x - 9$$

$$\text{So } x^2 + 5x - 2 = 3x^2 + 6x - 9$$

$$\text{Rearrange: } 2x^2 + x - 7 = 0$$

Use the formula with $a = 2$, $b = 1$, $c = -7$:

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times (-7)}}{2 \times 2}$$

$$x = \frac{-1 \pm \sqrt{57}}{4}$$

Answer: $\frac{-1 \pm \sqrt{57}}{4}$

Tip: Multiply the 3 on the right by the whole of $(x - 1)(x + 3)$. Expand $(x - 1)(x + 3)$ first, then multiply every term by 3.

Marks: 1 mark for multiplying by $(x - 1)(x + 3)$. 1 mark for $2x^2 + x - 7 = 0$. 1 mark for substituting into the formula correctly. 1 mark for $\frac{-1 \pm \sqrt{57}}{4}$.

7 (5 marks)

Solve $\frac{2}{x-4} + \frac{1}{x+3} = 1$

Give your answers in the form $p \pm q\sqrt{2}$ where p and q are integers.

Multiply every term by $(x - 4)(x + 3)$:

$$2(x + 3) + (x - 4) = (x - 4)(x + 3)$$

Left: $2x + 6 + x - 4 = 3x + 2$

Right: $x^2 + 3x - 4x - 12 = x^2 - x - 12$

So $3x + 2 = x^2 - x - 12$

Rearrange: $x^2 - 4x - 14 = 0$

Use the formula with $a = 1$, $b = -4$, $c = -14$:

$$x = \frac{4 \pm \sqrt{16 + 56}}{2} = \frac{4 \pm \sqrt{72}}{2}$$

Simplify the surd: $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

$$x = \frac{4 \pm 6\sqrt{2}}{2}$$

Divide each term on the top by 2: $x = 2 \pm 3\sqrt{2}$

Answer: $2 \pm 3\sqrt{2}$

Tip: When you simplify $\frac{4 \pm 6\sqrt{2}}{2}$, divide **both** terms on the top by 2, not just one of them.

Marks: 1 mark for multiplying by $(x - 4)(x + 3)$. 1 mark for $x^2 - 4x - 14 = 0$. 1 mark for using the formula correctly. 1 mark for $\sqrt{72} = 6\sqrt{2}$. 1 mark for $2 \pm 3\sqrt{2}$.
