

Equation of a Straight Line

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Find the gradient of the straight line that passes through the points (2, 5) and (6, 17)

$$\begin{aligned}\text{Gradient} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{17 - 5}{6 - 2} = \frac{12}{4} = 3\end{aligned}$$

Answer: 3

Tip: Put the change in y on top. Subtract the coordinates in the same order on the top and bottom.

Marks: 1 mark for $\frac{17-5}{6-2}$. 1 mark for 3.

2 (3 marks)

The straight line L has gradient -3
 L passes through the points (1, 5) and $(k, -10)$
Work out the value of k .

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{-10 - 5}{k - 1} = \frac{-15}{k - 1}$$

$$\text{So } \frac{-15}{k - 1} = -3$$

$$\text{Multiply both sides by } (k - 1): -15 = -3(k - 1)$$

$$\text{Divide by } -3: 5 = k - 1$$

$$\text{So } k = 6$$

Check: $\frac{-10-5}{6-1} = \frac{-15}{5} = -3 \checkmark$

Answer: $k = 6$

Tip: Keep the coordinates in the same order on the top and bottom of the gradient fraction:

$\frac{-10-5}{k-1}$, not $\frac{-10-5}{1-k}$.

Marks: 1 mark for $\frac{-10-5}{k-1}$ (or $y = -3x + 8$). 1 mark for $-15 = -3(k-1)$ (or $-10 = -3k + 8$). 1 mark for $k = 6$.

3 (3 marks)

Find an equation of the straight line that passes through the points $(-2, 7)$ and $(3, -3)$

$$\text{Gradient} = \frac{-3-7}{3-(-2)} = \frac{-10}{5} = -2$$

So the line is $y = -2x + c$

Substitute $(-2, 7)$: $7 = -2 \times (-2) + c$

$$7 = 4 + c, \text{ so } c = 3$$

Check with $(3, -3)$: $-2 \times 3 + 3 = -3 \checkmark$

Answer: $y = -2x + 3$

Marks: 1 mark for the gradient. 1 mark for substituting a point. 1 mark for $y = -2x + 3$.

4 (3 marks)

The line L has equation $3x + 2y = 8$

Find the gradient of L and the coordinates of the point where L crosses the y -axis.

Rearrange into the form $y = mx + c$.

Subtract $3x$: $2y = -3x + 8$

Divide **every** term by 2: $y = -\frac{3}{2}x + 4$

The gradient is the number in front of x : $-\frac{3}{2}$

The line crosses the y -axis where $x = 0$, which is at $y = 4$.

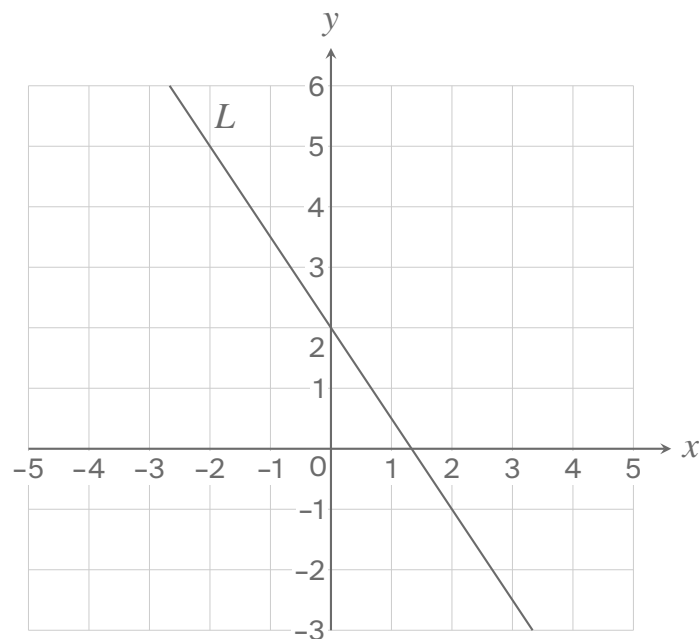
Answer: Gradient $-\frac{3}{2}$; crosses the y -axis at $(0, 4)$

Tip: The gradient is not 3 or -3 : you must make y the subject first. Give the y -intercept as a point, $(0, 4)$.

Marks: 1 mark for rearranging to $y = \dots$ 1 mark for the gradient $-\frac{3}{2}$. 1 mark for $(0, 4)$.

5 (4 marks)

The straight line L is drawn on the grid.



(a) Find an equation for L .

L crosses the y -axis at $(0, 2)$, so $c = 2$.

L also passes through $(2, -1)$.

$$\text{Gradient} = \frac{-1 - 2}{2 - 0} = \frac{-3}{2} = -\frac{3}{2}$$

The line slopes downwards, so a negative gradient makes sense.

$$\text{So } y = -\frac{3}{2}x + 2$$

Answer: $y = -\frac{3}{2}x + 2$

Tip: Your answer must be an equation, starting $y = \dots$. Giving coordinates, or writing $L = \dots$, does not get the final mark. Check the sign: L goes down from left to right.

Marks: 1 mark for the gradient $-\frac{3}{2}$. 1 mark for the y-intercept 2. 1 mark for $y = -\frac{3}{2}x + 2$.

- (b) The line N is parallel to L and passes through the point $(0, -4)$
Write down an equation for N .

Parallel lines have the same gradient, so N has gradient $-\frac{3}{2}$.

N crosses the y-axis at $(0, -4)$, so $c = -4$.

Answer: $y = -\frac{3}{2}x - 4$

6 (3 marks)

A is the point $(4, 1)$ and B is the point $(10, 5)$

M is the midpoint of AB .

Find the equation of the line through M with gradient 3

Midpoint: find the mean of the x -coordinates and the mean of the y -coordinates.

$$M = \left(\frac{4+10}{2}, \frac{1+5}{2} \right) = (7, 3)$$

The line is $y = 3x + c$

Substitute $(7, 3)$: $3 = 3 \times 7 + c$

$$3 = 21 + c, \text{ so } c = -18$$

Answer: $y = 3x - 18$

Marks: 1 mark for $M(7, 3)$. 1 mark for substituting M into $y = 3x + c$. 1 mark for $y = 3x - 18$.

7 (2 marks)

Show that the point (5,12) lies on the line through (1,4) and (3,8)

$$\text{Gradient} = \frac{8-4}{3-1} = \frac{4}{2} = 2$$

Substitute (1,4) into $y = 2x + c$: $4 = 2 + c$, so $c = 2$

The line is $y = 2x + 2$

When $x = 5$: $y = 2 \times 5 + 2 = 12$

So the point (5,12) lies on the line.

Answer: The line is $y = 2x + 2$, and when $x = 5$, $y = 12$.

Marks: 1 mark for the equation of the line. 1 mark for substituting $x = 5$ with a conclusion.

8 (5 marks)

In the diagram, A is the point $(-2,1)$ and B is the point $(10,7)$

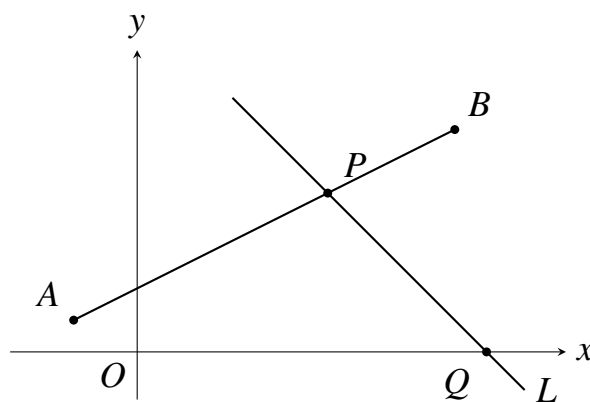


Diagram **NOT** accurately drawn

P is the point on AB such that $AP:PB = 2:1$

The line L passes through P and has gradient -1

L crosses the x -axis at the point Q .

Find the coordinates of Q .

$AP:PB = 2:1$ means P is $\frac{2}{3}$ of the way from A to B .

From A to B : x goes up by $10 - (-2) = 12$ and y goes up by $7 - 1 = 6$.

$\frac{2}{3}$ of this: x up by 8, y up by 4

So $P = (-2 + 8, 1 + 4) = (6, 5)$

L has gradient -1 , so $y = -x + c$

Substitute $P(6, 5)$: $5 = -6 + c$, so $c = 11$

L is $y = -x + 11$

On the x -axis $y = 0$: $0 = -x + 11$, so $x = 11$

Answer: $Q(11, 0)$

Tip: A ratio of 2:1 splits AB into $2 + 1 = 3$ equal parts, so P is $\frac{2}{3}$ of the way from A — not $\frac{1}{2}$.

Marks: 1 mark for a method to find P . 1 mark for $P(6, 5)$. 1 mark for substituting P into $y = -x + c$. 1 mark for $y = -x + 11$. 1 mark for $(11, 0)$.
