

Equation of a Circle and Tangents

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

The equation of a circle is $x^2 + y^2 = 20.25$

Find the radius of the circle.

A circle with centre (0,0) and radius r has equation $x^2 + y^2 = r^2$.

So $r^2 = 20.25$

$r = \sqrt{20.25} = 4.5$

Answer: 4.5

2 (1 mark)

The equation of a circle is $x^2 + y^2 = 64$

Write down the coordinates of the points where the circle crosses the x -axis.

$r^2 = 64$, so the radius is 8.

The centre is (0,0), so the circle crosses the x -axis 8 to the right and 8 to the left of the origin.

Answer: (8, 0) and (−8, 0)

3 (2 marks)

The point $P(-2, 6)$ lies on a circle with centre the origin.

Find the equation of the circle.

The radius is the distance from O to P .

By Pythagoras: $r^2 = (-2)^2 + 6^2 = 4 + 36 = 40$

The equation is $x^2 + y^2 = r^2$.

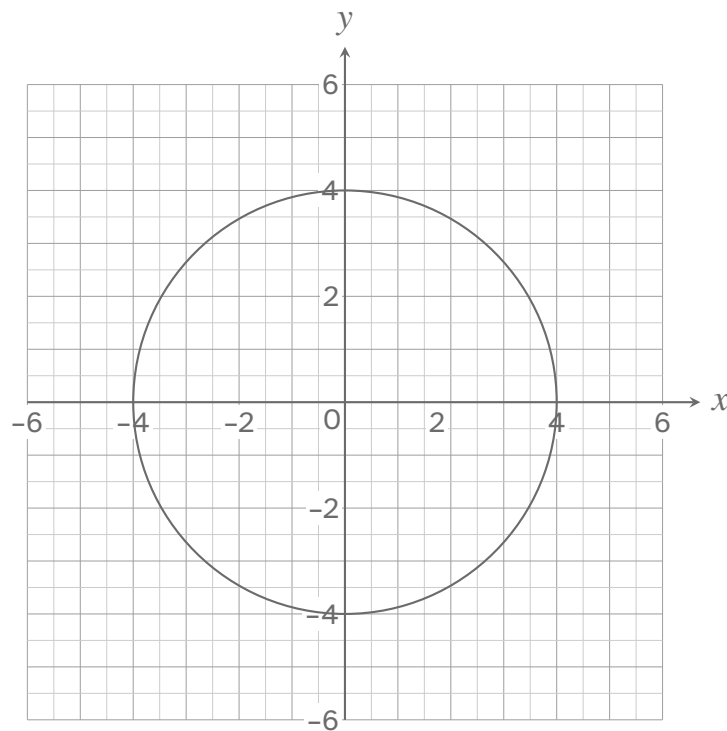
Answer: $x^2 + y^2 = 40$

Tip: You need r^2 , which is 40. There is no need to find $r = \sqrt{40}$, and $x^2 + y^2 = \sqrt{40}$ is wrong.

Marks: 1 mark for $(-2)^2 + 6^2$. 1 mark for $x^2 + y^2 = 40$.

4 (5 marks)

The graph of a circle with centre O is drawn on the grid.



(a) Write down an equation of the circle.

The centre is $O(0,0)$.

From the grid, the circle crosses the x -axis at 4 and -4 , so the radius is 4.

$$x^2 + y^2 = r^2 \text{ gives } x^2 + y^2 = 16$$

Answer: $x^2 + y^2 = 16$

Tip: The number on the right is the radius **squared**: $4^2 = 16$, not 4.

- (b) By drawing a suitable straight line on the grid, find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 16$$

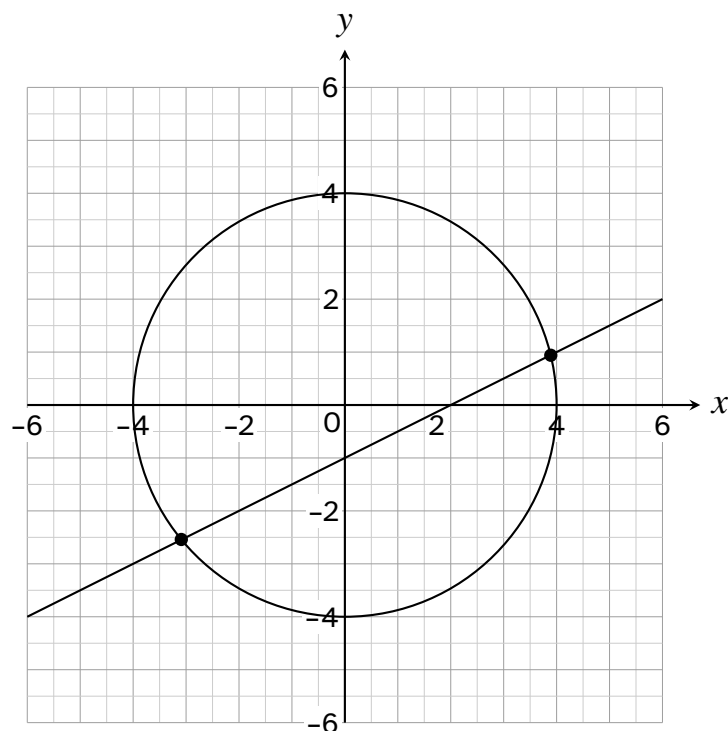
$$x - 2y = 2$$

Draw the line $x - 2y = 2$ on the grid.

When $y = 0$, $x = 2$, and when $x = 0$, $y = -1$.

So the line goes through $(2, 0)$ and $(0, -1)$.

Read off the two points where the line crosses the circle.



Answer: $x \approx 3.9, y \approx 0.9$ and $x \approx -3.1, y \approx -2.5$

Tip: Each solution is a pair: an x value **and** the matching y value. Draw the line accurately with a ruler through two points you have worked out.

Marks: 1 mark for drawing the line $x - 2y = 2$. 1 mark for one correct pair. 1 mark for both pairs (answers within 0.2 are accepted).

(c) Explain why the simultaneous equations

$$x^2 + y^2 = 16$$

$$y = 5$$

have no solutions.

$y = 5$ is a horizontal line 5 units above the x -axis.

The highest point of the circle is $(0, 4)$.

So the line $y = 5$ is above the circle and never meets it.

Answer: The line $y = 5$ does not meet the circle, because the circle only reaches up to $y = 4$

5 (4 marks)

$P(-5, 12)$ is a point on the circle $x^2 + y^2 = 169$

Find an equation of the tangent to the circle at P .

Give your answer in the form $ax + by = c$ where a , b and c are integers.

$$\text{Gradient of radius } OP = \frac{12 - 0}{-5 - 0} = -\frac{12}{5}$$

The tangent is perpendicular to the radius.

$$\text{Tangent gradient} = \frac{5}{12} \text{ (negative reciprocal)}$$

$$\text{Use } y - y_1 = m(x - x_1) \text{ with } P(-5, 12): y - 12 = \frac{5}{12}(x + 5)$$

$$\text{Multiply both sides by 12: } 12y - 144 = 5x + 25$$

$$\text{Rearrange: } 5x - 12y = -169$$

$$\text{Check with } P: 5 \times (-5) - 12 \times 12 = -25 - 144 = -169 \quad \checkmark$$

Answer: $5x - 12y = -169$

Tip: Give the answer in the form asked for, with whole numbers: multiply by 12 to clear the fraction. Keep the sign of the gradient right: $+\frac{5}{12}$.

Marks: 1 mark for the gradient of OP . 1 mark for the tangent gradient. 1 mark for a correct equation of the tangent. 1 mark for $5x - 12y = -169$ (or any correct integer form).

6 (4 marks)

The point $P(1, 6)$ lies on the circle $x^2 + y^2 = 37$

The tangent to the circle at P passes through the point $Q(k, 5)$

Find the value of k .

You must show all your working.

$$\text{Gradient of radius } OP = \frac{6}{1} = 6$$

The tangent is perpendicular to OP , so its gradient is $-\frac{1}{6}$.

$$\text{Use } y - y_1 = m(x - x_1) \text{ with } P(1, 6): y - 6 = -\frac{1}{6}(x - 1)$$

$$\text{Multiply by 6: } 6y - 36 = -x + 1, \text{ so the tangent is } x + 6y = 37$$

$Q(k, 5)$ is on the tangent: $k + 6 \times 5 = 37$

$k + 30 = 37$, so $k = 7$

Answer: $k = 7$

Tip: The tangent is perpendicular to the **radius** OP , not to the line OQ . Find the tangent's equation first, then substitute $y = 5$.

Marks: 1 mark for the gradient of OP . 1 mark for the tangent gradient $-\frac{1}{6}$. 1 mark for a correct equation of the tangent. 1 mark for $k = 7$.

7 (4 marks)

The point $P(a, a - 1)$ lies on the circle $x^2 + y^2 = 41$, where $a > 0$

Find an equation of the tangent to the circle at P .

Give your answer in the form $px + qy = r$ where p , q and r are integers.

P is on the circle: $a^2 + (a - 1)^2 = 41$

Expand: $a^2 + a^2 - 2a + 1 = 41$, so $2a^2 - 2a - 40 = 0$

Divide by 2: $a^2 - a - 20 = 0$

Factorise: $(a - 5)(a + 4) = 0$, so $a = 5$ ($a > 0$)

So P is $(5, 4)$.

Gradient of radius $OP = \frac{4}{5}$, so the tangent gradient is $-\frac{5}{4}$.

$$y - 4 = -\frac{5}{4}(x - 5)$$

Multiply by 4: $4y - 16 = -5x + 25$

Rearrange: $5x + 4y = 41$

Answer: $5x + 4y = 41$

Tip: $(a - 1)^2 = a^2 - 2a + 1$, not $a^2 + 1$. Use $a > 0$ to reject $a = -4$, then find both coordinates of P before working out any gradients.

Marks: 1 mark for $a^2 + (a - 1)^2 = 41$. 1 mark for $a = 5$. 1 mark for the tangent gradient $-\frac{5}{4}$. 1 mark for $5x + 4y = 41$ (or any correct integer form).

8 (5 marks)

The line $y = x + 2$ intersects the circle $x^2 + y^2 = 34$ at two points.
Find the coordinates of the two points.

Substitute $y = x + 2$ into the circle: $x^2 + (x + 2)^2 = 34$

Expand: $x^2 + x^2 + 4x + 4 = 34$

$$2x^2 + 4x - 30 = 0$$

Divide by 2: $x^2 + 2x - 15 = 0$

Factorise: $(x + 5)(x - 3) = 0$

So $x = 3$ or $x = -5$

Find y using $y = x + 2$:

$$x = 3: y = 5$$

$$x = -5: y = -3$$

Answer: (3, 5) and (-5, -3)

Marks: 1 mark for substituting. 1 mark for expanding. 1 mark for solving. 1 mark for both x values. 1 mark for both points.
