

Direct and Inverse Proportion - Forming Equations

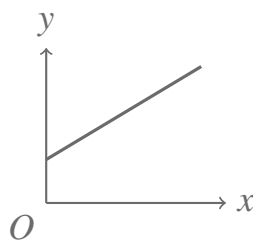
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

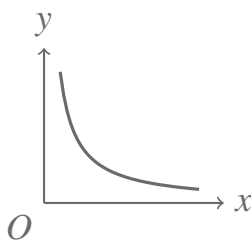
Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

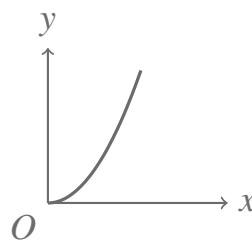
Here are sketches of four graphs of y against x .



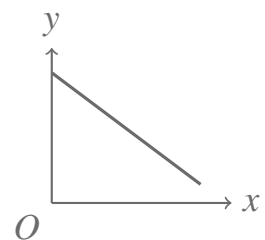
Graph P



Graph Q



Graph R



Graph S

- (a) Write down the letter of the graph that could show y is inversely proportional to x .

Inverse proportion: $y = \frac{k}{x}$

As x increases, y decreases, getting closer and closer to the x -axis without reaching it.

The graph is a curve, so it is Graph Q (not the straight line S).

Answer: Graph Q

Tip: Inverse proportion is a curve, not a straight line sloping downwards. Graph S would reach $y = 0$.

- (b) Kai says, “Graph P shows that y is directly proportional to x because it is a straight line.”

Is Kai correct?

You must give a reason for your answer.

Direct proportion: $y = kx$, so the graph is a straight line that goes **through the origin**.

Graph P is a straight line, but it does not pass through the origin O .

So Kai is not correct.

Answer: No: the graph of direct proportion must pass through the origin, and Graph P does not.

2 (3 marks)

w is directly proportional to the square of v .

Complete the table of values.

v	2	5	
w	12		147

w is directly proportional to v^2 : $w = kv^2$

Substitute $v = 2$ and $w = 12$: $12 = k \times 4$, so $k = 3$

So $w = 3v^2$

When $v = 5$: $w = 3 \times 25 = 75$

When $w = 147$: $147 = 3v^2$, so $v^2 = 49$

$v = 7$ (the other values of v in the table are positive)

Answer: 75 and 7

Tip: For the last box, divide by 3 and then take the **square root**: $v^2 = 49$ gives $v = 7$, not 49.

Marks: 1 mark for $k = 3$ (or $w = 3v^2$). 1 mark for 75. 1 mark for 7.

3 (5 marks)

T is directly proportional to the square root of L .

When $L = 9$, $T = 6$

- (a) Find a formula for T in terms of L .

T is directly proportional to the square root of L : $T = k\sqrt{L}$

Substitute $L = 9$ and $T = 6$: $6 = k \times \sqrt{9} = 3k$

$$k = 2$$

$$\text{So } T = 2\sqrt{L}$$

Answer: $T = 2\sqrt{L}$

Marks: 1 mark for $T = k\sqrt{L}$. 1 mark for $k = 2$. 1 mark for $T = 2\sqrt{L}$.

- (b) Find L when $T = 10$

$$10 = 2\sqrt{L}$$

$$\sqrt{L} = 5$$

Square both sides: $L = 25$

Answer: 25

Marks: 1 mark for $\sqrt{L} = 5$. 1 mark for 25.

4 (3 marks)

The time, t hours, taken to build a wall is inversely proportional to the number of workers, n .

6 workers take 10 hours.

How long will 4 workers take?

Inversely proportional: $t = \frac{k}{n}$

6 workers take 10 hours: $10 = \frac{k}{6}$, so $k = 60$

$$\text{So } t = \frac{60}{n}$$

When $n = 4$: $t = 60 \div 4 = 15$ hours

Answer: 15 hours

Tip: Fewer workers take **longer**. If your answer is less than 10 hours, you have used direct proportion by mistake.

Marks: 1 mark for $k = 60$ (or 6×10). 1 mark for $60 \div 4$. 1 mark for 15.

5 (3 marks)

f is inversely proportional to s^2

When $s = 2$, $f = 12$

Find the value of f when $s = 4$

f is inversely proportional to s^2 : $f = \frac{k}{s^2}$

Substitute $s = 2$ and $f = 12$: $12 = \frac{k}{4}$, so $k = 48$

$$\text{So } f = \frac{48}{s^2}$$

$$\text{When } s = 4: f = \frac{48}{16} = 3$$

Answer: 3

Tip: Put s^2 on the **bottom**: $f = \frac{k}{s^2}$, not $f = ks^2$.

Marks: 1 mark for $f = \frac{k}{s^2}$. 1 mark for $k = 48$. 1 mark for 3.

6 (2 marks)

Q is inversely proportional to the square root of R .

$Q = 15$ when $R = 16$

Show that $Q = 10$ when $R = 36$

$$Q = \frac{k}{\sqrt{R}}$$

$$Q = 15 \text{ when } R = 16: 15 = \frac{k}{\sqrt{16}} = \frac{k}{4}, \text{ so } k = 60$$

$$\text{So } Q = \frac{60}{\sqrt{R}}$$

$$\text{When } R = 36: Q = \frac{60}{\sqrt{36}} = \frac{60}{6} = 10 \quad \checkmark$$

Answer: Shown

Marks: 1 mark for $k = 60$. 1 mark for a complete method that reaches 10.

7 (3 marks)

y is inversely proportional to x^2

x is increased by 25%

Work out the percentage decrease in y .

$$y = \frac{k}{x^2}$$

x is increased by 25%, so x is multiplied by 1.25

Then x^2 is multiplied by $1.25^2 = 1.5625$

x^2 is on the bottom, so y is multiplied by $\frac{1}{1.5625} = 0.64$

0.64 is 64% of the original value.

Percentage decrease = $100 - 64 = 36\%$

Answer: 36%

Tip: Divide by 1.25^2 because x^2 is on the bottom of the fraction. The answer is not 25% or 56.25%.

Marks: 1 mark for 1.25^2 (or 1.5625). 1 mark for 0.64. 1 mark for 36.

8 (4 marks)

w is directly proportional to v^2

$w = 18$ when $v = 3$

v is inversely proportional to $\sqrt{u}$

$v = 2$ when $u = 36$

Find the value of u when $w = 32$

You must show all your working.

w is directly proportional to v^2 : $w = kv^2$

$18 = k \times 9$, so $k = 2$ and $w = 2v^2$

v is inversely proportional to $\sqrt{u}$: $v = \frac{c}{\sqrt{u}}$

$2 = \frac{c}{\sqrt{36}} = \frac{c}{6}$, so $c = 12$ and $v = \frac{12}{\sqrt{u}}$

When $w = 32$: $32 = 2v^2$, so $v^2 = 16$

$v^2 = \frac{144}{u}$, so $\frac{144}{u} = 16$

$u = 144 \div 16 = 9$

Answer: 9

Tip: Use two different constants (k and c). Work with v^2 : squaring $v = \frac{12}{\sqrt{u}}$ gives $v^2 = \frac{144}{u}$, which avoids square roots.

Marks: 1 mark for $w = kv^2$ with $k = 2$. 1 mark for $v = \frac{c}{\sqrt{u}}$ with $c = 12$. 1 mark for $v^2 = 16$ (or $w = \frac{288}{u}$). 1 mark for 9.