

Cumulative Frequency

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (5 marks)

The cumulative frequency table gives information about the journey times, in minutes, of 80 commuters.

Time (t minutes)	$10 < t \leq 20$	$10 < t \leq 30$	$10 < t \leq 40$	$10 < t \leq 50$	$10 < t \leq 60$	$10 < t \leq 70$
Cumulative frequency	5	18	42	61	74	80

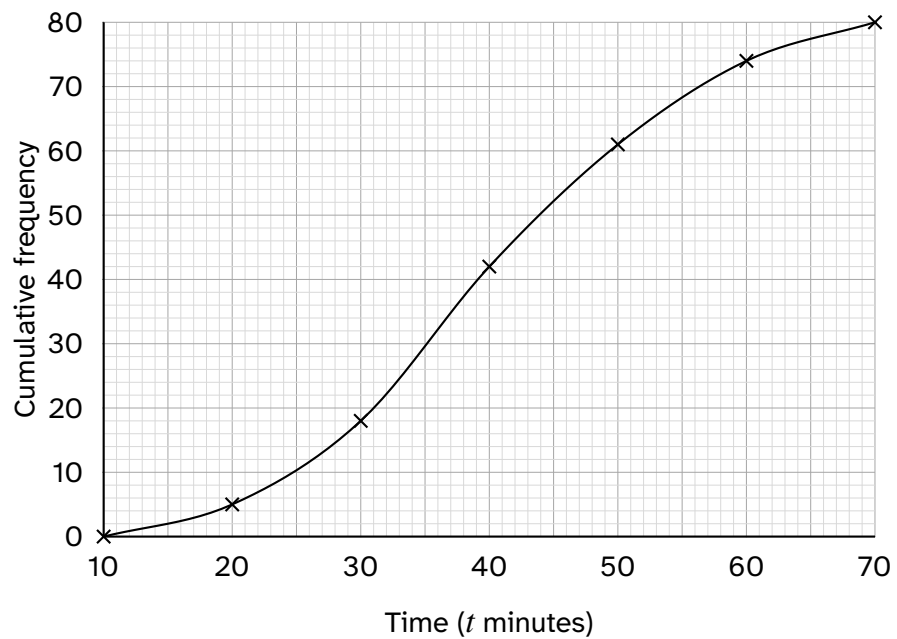
(a) On the grid, draw a cumulative frequency graph for this information.

Plot each cumulative frequency at the upper end of its class:

(20, 5), (30, 18), (40, 42), (50, 61), (60, 74), (70, 80)

The times start at 10 minutes, so also plot (10, 0).

Join the points with a smooth curve.



Answer: See graph

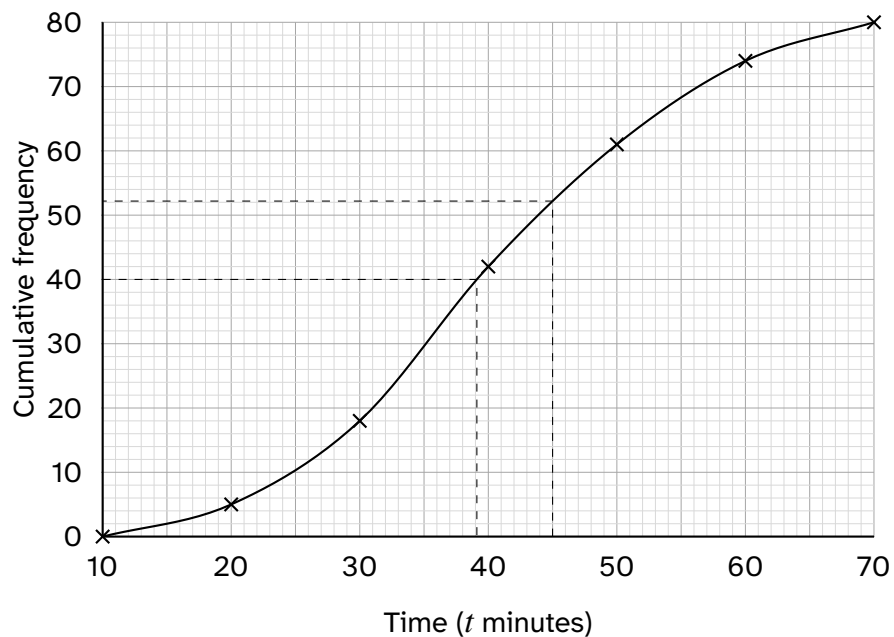
Tip: Plot at the upper class boundaries (20, 30, 40, ...), not at the midpoints. Start the curve at (10, 0), not at (0, 0).

Marks: 1 mark for at least 5 points plotted correctly. 1 mark for a fully correct curve (or straight lines).

- (b) Use your graph to find an estimate for the median time.

Half of 80 is 40.

Read across from 40 to the curve, then down.



This gives about 39 minutes.

Answer: 39 (accept 38 to 40)

- (c) Use your graph to find an estimate for the number of commuters whose journey took longer than 45 minutes.

Read up from 45 minutes to the curve, then across: about 52.

(This reading is shown on the graph in part (b).)

So about 52 commuters took 45 minutes or less.

Longer than 45 minutes: $80 - 52 = 28$

Answer: 28 (accept 25 to 31)

Tip: The graph gives the number who took 45 minutes **or less**. For “longer than”, subtract your reading from 80.

Marks: 1 mark for reading the graph at 45 minutes. 1 mark for $80 -$ your reading (answer from 25 to 31).

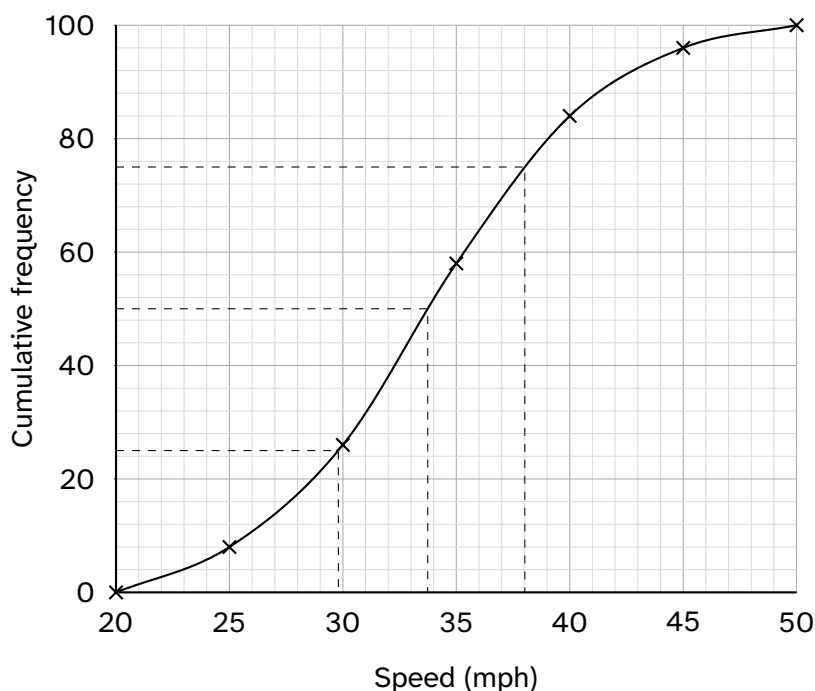
2 (6 marks)

The cumulative frequency graph shows information about the speeds of 100 cars on a road.

- (a) Use the graph to find an estimate for the median speed.

Half of 100 is 50.

Read across from 50 to the curve, then down: about 34 mph.



Answer: 34 (accept 33 to 35)

Tip: Read across from half the total frequency (50). Do not read from the middle of the speed axis.

- (b) Use the graph to find an estimate for the interquartile range.

Lower quartile: $\frac{1}{4}$ of 100 is 25. Read across from 25: LQ $\approx$ 30 mph.

Upper quartile: $\frac{3}{4}$ of 100 is 75. Read across from 75: UQ $\approx$ 38 mph.

(These readings are shown on the graph in part (a).)

$$\text{IQR} \approx 38 - 30 = 8 \text{ mph}$$

Answer: 8 (accept 7 to 9.5)

Tip: The quartiles are at $\frac{1}{4}$ and $\frac{3}{4}$ of the total frequency (25 and 75). Subtract the two **speeds**, not $75 - 25$.

Marks: 1 mark for reading the quartiles at 25 and 75. 1 mark for an answer from 7 to 9.5.

- (c) The speed limit on the road is 40 mph.
Find an estimate for the percentage of these cars that were travelling faster than the speed limit.

Read up from 40 mph to the curve: 84 cars were travelling at 40 mph or less.

Faster than 40 mph: $100 - 84 = 16$ cars

As a percentage: $\frac{16}{100} \times 100 = 16\%$

Answer: 16% (accept 15% to 17%)

Tip: The graph gives the number of cars at 40 mph **or less**. Subtract from 100 to find the number that were faster.

Marks: 1 mark for reading 84 at 40 mph. 1 mark for $100 - 84 = 16$. 1 mark for 16%.

3 (1 mark)

The table gives information about the times, in minutes, 50 people took to finish a crossword.

Time (t minutes)	Frequency
$0 < t \leq 10$	7
$10 < t \leq 20$	14
$20 < t \leq 30$	16
$30 < t \leq 40$	9
$40 < t \leq 50$	4

Kim is going to draw a cumulative frequency graph for this information.

For the class $20 < t \leq 30$ she plots the point (25, 37)

Explain what is wrong with this point.

The cumulative frequency $7 + 14 + 16 = 37$ is correct.

But it should be plotted at the **upper** class boundary, 30, not at the midpoint 25.

The correct point is (30, 37).

Answer: It should be plotted at the upper class boundary: (30, 37), not at the midpoint 25

Tip: A cumulative frequency counts everything **up to the end** of the class, so it is plotted at the upper class boundary.

4 (6 marks)

The table gives information about the masses of 120 parcels.

Mass (m kg)	Frequency
$0 < m \leq 2$	8
$2 < m \leq 4$	22
$4 < m \leq 6$	36
$6 < m \leq 8$	30
$8 < m \leq 10$	16
$10 < m \leq 12$	8

(a) Complete the cumulative frequency table.

Keep a running total of the frequencies:

8, $8 + 22 = 30$, $30 + 36 = 66$, $66 + 30 = 96$, $96 + 16 = 112$, $112 + 8 = 120$

Mass (m kg)	Cumulative frequency
$0 < m \leq 2$	8
$0 < m \leq 4$	30
$0 < m \leq 6$	66
$0 < m \leq 8$	96
$0 < m \leq 10$	112
$0 < m \leq 12$	120

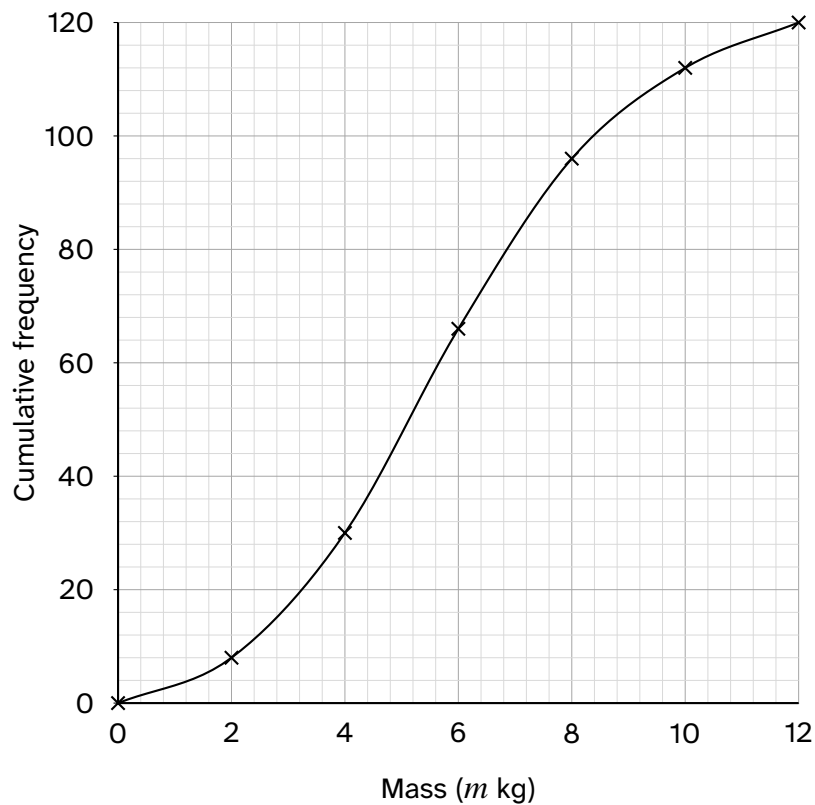
Answer: 8, 30, 66, 96, 112, 120

- (b) On the grid, draw a cumulative frequency graph for your completed table.

Plot at the upper class boundaries: (2, 8), (4, 30), (6, 66), (8, 96), (10, 112), (12, 120)

The first class starts at 0 kg, so start the graph at (0, 0).

Join with a smooth curve.



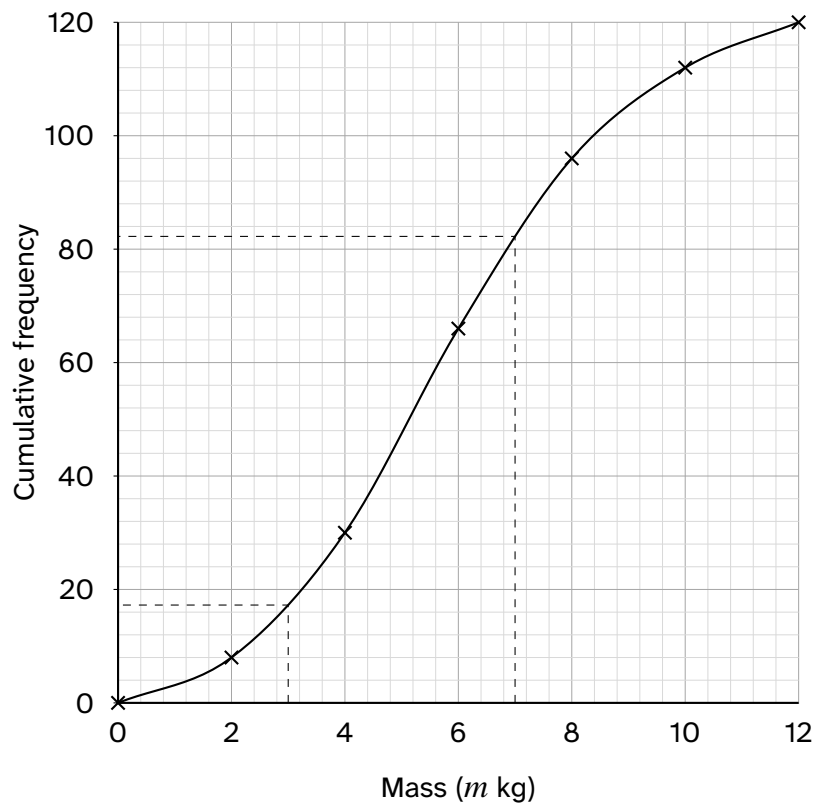
Answer: See graph

Marks: 1 mark for at least 5 points plotted correctly. 1 mark for a fully correct curve (or straight lines).

- (c) Use your graph to find an estimate for the percentage of the parcels with a mass between 3 kg and 7 kg.

Read up from 3 kg to the curve, then across: about 17 parcels.

Read up from 7 kg: about 82 parcels.



Between 3 kg and 7 kg: $82 - 17 = 65$ parcels

As a percentage: $\frac{65}{120} \times 100 = 54.2\%$ (1 dp)

Answer: 54.2% (accept about 50% to 58%)

Tip: Subtract the two readings to find how many parcels are between 3 kg and 7 kg, then divide by 120, the total.

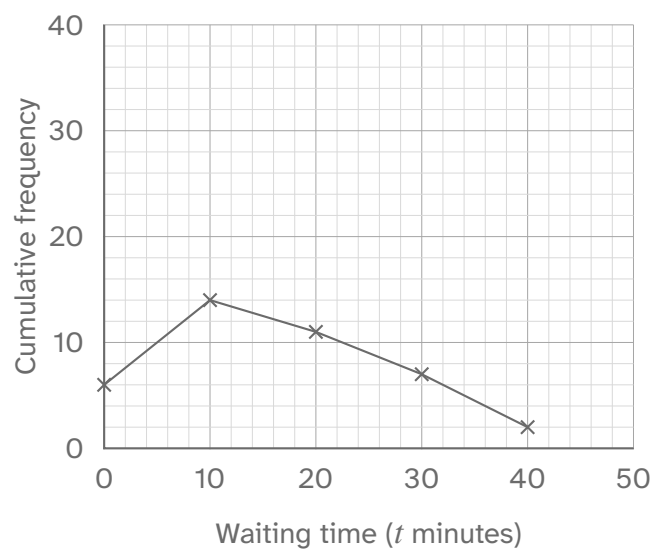
Marks: 1 mark for reading the graph at 3 kg or 7 kg. 1 mark for the difference of your two readings. 1 mark for the correct percentage from your readings (about 50% to 58%).

5 (2 marks)

The table gives information about the times, in minutes, that 40 patients waited at a health centre.

Time (t minutes)	Frequency
$0 < t \leq 10$	6
$10 < t \leq 20$	14
$20 < t \leq 30$	11
$30 < t \leq 40$	7
$40 < t \leq 50$	2

Ella draws this graph for the information in the table.



Write down **two** different things that are wrong with Ella's graph.

1. Ella has plotted the frequencies (6, 14, 11, ...), not the cumulative frequencies (6, 20, 31, 38, 40).

2. She has plotted the points at the lower class boundaries (0, 10, 20, ...), not the upper class boundaries (10, 20, 30, ...).

Answer: Frequencies plotted instead of cumulative frequencies; points plotted at the lower (not upper) class boundaries

Tip: A cumulative frequency graph can never go down: if it does, the running totals have not been used.

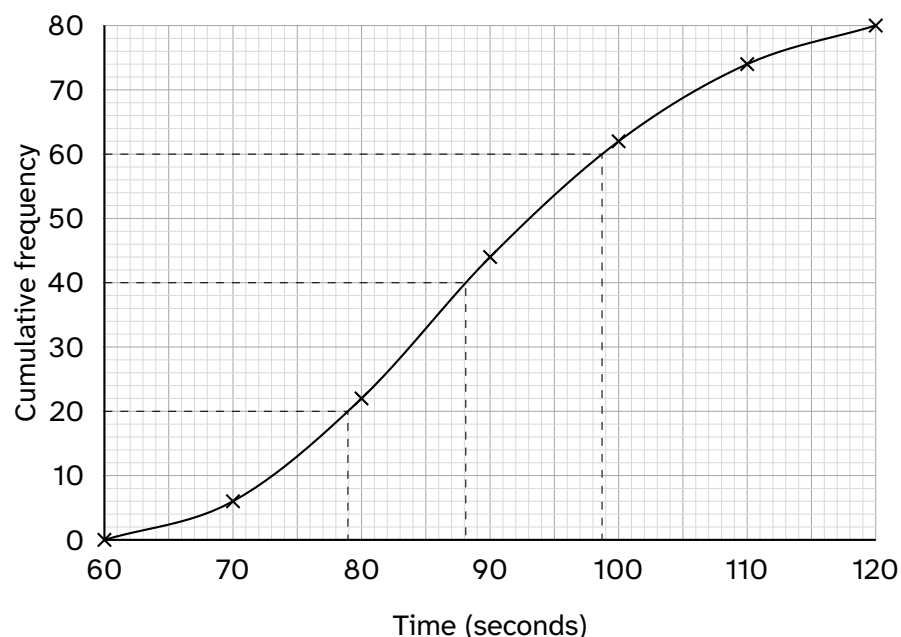
6 (5 marks)

The cumulative frequency graph gives information about the times, in seconds, that 80 students took to run 400 m. The fastest student took 63 seconds. The slowest student took 117 seconds.

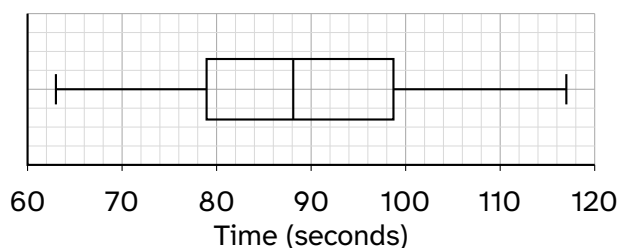
(a) On the grid below, draw a box plot for the distribution of the times.

There are 80 students, so read across from 20, 40 and 60 (a quarter, half and three quarters of 80).

LQ $\approx$ 79, median $\approx$ 88, UQ $\approx$ 99



Draw the box from 79 to 99 with a line at 88, and whiskers out to 63 and 117.



Answer: 63, 79, 88, 99, 117

Tip: Read the quartiles from the cumulative frequency axis at 20, 40 and 60. Do not use quarters of the time scale.

Marks: 1 mark for the median (87 to 89). 1 mark for both quartiles (78 to 80 and 98 to 100). 1 mark for a box plot with whiskers to 63 and 117.

(b) 80 adults also ran 400 m.

For the adults

the median time was 95 seconds

the interquartile range was 26 seconds.

Compare the distribution of the times of the students with the distribution of the times of the adults.

Average: the students' median (about 88 seconds) is lower than the adults' median (95 seconds).

So on average the students ran faster (took less time).

Spread: the students' IQR (about $99 - 79 = 20$ seconds) is smaller than the adults' IQR (26 seconds).

So the students' times were more consistent.

Answer: Students faster on average (median $88 < 95$ s); students' times more consistent (IQR about $20 < 26$ s)

Tip: A lower time means **faster**. Make one comparison of an average and one of a spread, and say what each means for the runners.

Marks: 1 mark for comparing the medians, in context. 1 mark for comparing the interquartile ranges, in context.