

Conditional Probability

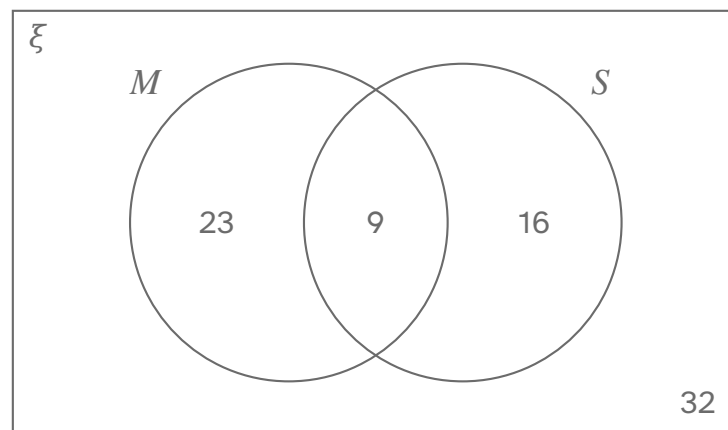
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

80 students were asked if they play a musical instrument (M) or sing in a choir (S).
The Venn diagram shows the results.



- (a) One of the students is chosen at random.
Write down the probability that this student does not play an instrument and does not sing in a choir.

Neither: the region outside both circles, which has 32 students.

The student is chosen from all 80 students.

Answer: $\frac{32}{80}$

- (b) One of the students who sing in a choir is chosen at random.
Work out the probability that this student also plays an instrument.

The student is chosen from the choir singers only.

Students who sing in a choir (all of circle S): $9 + 16 = 25$

9 of them also play an instrument.

Answer: $\frac{9}{25}$

Tip: “One of the students who sing in a choir” means the bottom of the fraction is 25, not 80.

Marks: 1 mark for 25 as the denominator. 1 mark for $\frac{9}{25}$.

2 (3 marks)

There are 12 sweets in a bag: 5 are toffees and 7 are mints.

Ali takes a sweet at random and eats it. He then takes another sweet at random and eats it.

Work out the probability that he eats at least one toffee.

“At least one toffee” is everything except “no toffees”.

No toffee means mint, then mint.

The sweets are eaten, so the second pick is out of 11.

$$P(\text{mint, mint}) = \frac{7}{12} \times \frac{6}{11} = \frac{42}{132}$$

$$P(\text{at least one toffee}) = 1 - \frac{42}{132} = \frac{90}{132}$$

Answer: $\frac{90}{132} = \frac{15}{22}$

Tip: “At least one” includes two toffees. Using $1 - P(\text{no toffee})$ means you cannot miss a case.

Marks: 1 mark for $\frac{7}{12} \times \frac{6}{11}$ (or one correct product such as $\frac{5}{12} \times \frac{7}{11}$). 1 mark for $1 - \frac{42}{132}$ or adding all three toffee cases. 1 mark for $\frac{90}{132}$ or an equivalent.

3 (5 marks)

The probability that it rains on Saturday is 0.3

If it rains on Saturday, the probability that it rains on Sunday is 0.6

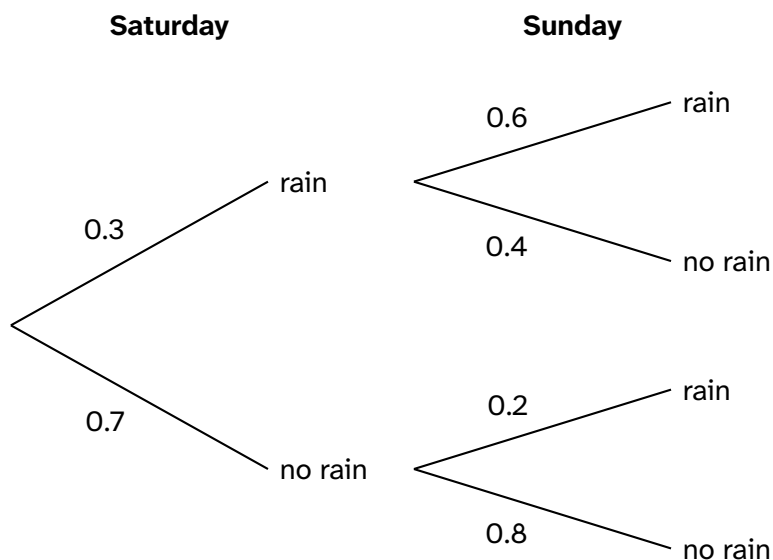
If it does not rain on Saturday, the probability that it rains on Sunday is 0.2

(a) Complete the probability tree diagram.

No rain on Saturday: $1 - 0.3 = 0.7$

After rain on Saturday: rain 0.6, no rain $1 - 0.6 = 0.4$

After no rain on Saturday: rain 0.2, no rain $1 - 0.2 = 0.8$



Answer: 0.7; 0.6, 0.4; 0.2, 0.8

Marks: 1 mark for 0.7. 1 mark for all four Sunday probabilities.

(b) Work out the probability that it rains on exactly one of the two days.

Rain on exactly one day can happen in two ways.

Rain Saturday, no rain Sunday: $0.3 \times 0.4 = 0.12$

No rain Saturday, rain Sunday: $0.7 \times 0.2 = 0.14$

Add: $0.12 + 0.14 = 0.26$

Answer: 0.26

Marks: 1 mark for 0.3×0.4 or 0.7×0.2 . 1 mark for adding both products. 1 mark for 0.26.

4 (5 marks)

Ruby plays two matches of table tennis against Alex.

The probability that Ruby wins the first match is 0.55

If Ruby wins the first match, the probability that she does not win the second is 0.3

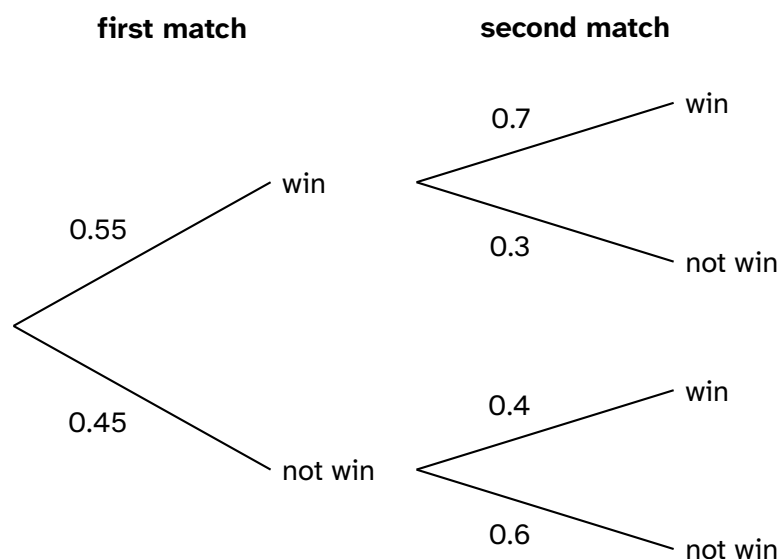
If Ruby does not win the first match, the probability that she wins the second is 0.4

(a) Complete the probability tree diagram for this information.

First match: win 0.55, not win $1 - 0.55 = 0.45$

After winning the first: not win the second is 0.3, so win is $1 - 0.3 = 0.7$

After not winning the first: win the second is 0.4, so not win is $1 - 0.4 = 0.6$



Answer: 0.55, 0.45; 0.7, 0.3; 0.4, 0.6

Tip: Read each sentence carefully: one gives the probability of **not** winning, the other of winning. Check each pair adds up to 1.

Marks: 1 mark for the first pair. 1 mark for both second pairs.

- (b) Ruby won exactly one of the two matches.
Work out the probability that it was the first match that she won.
You must show your working.

Won the first only: win, then not win: $0.55 \times 0.3 = 0.165$

Won the second only: not win, then win: $0.45 \times 0.4 = 0.18$

Exactly one win: $0.165 + 0.18 = 0.345$

We know she won exactly one, so divide:

$$\frac{0.165}{0.345} = \frac{165}{345} = \frac{11}{23}$$

Answer: $\frac{11}{23}$ (0.478...)

Tip: 0.165 is the probability that she wins only the first match out of **all** possible outcomes. Given that she won exactly one, divide by 0.345.

Marks: 1 mark for 0.55×0.3 and 0.45×0.4 . 1 mark for 0.345. 1 mark for $\frac{11}{23}$ or an equivalent.

5 (6 marks)

There are 120 students in a year group.

Some belong to the football club (F), the gym club (G) or the netball club (N).

6 students belong to all three clubs.

10 students belong to the football club and the gym club but not the netball club.

13 students belong to the gym club and the netball club.

45 students belong to the football club.

41 students belong to the gym club.

37 students belong to the netball club.

30 students do not belong to any of these clubs.

- (a) Complete the Venn diagram for this information.

Start in the middle: all three = 6

F and G but not N : 10

G and N only: $13 - 6 = 7$ (the 13 includes the 6 in all three)

G only: $41 - 10 - 6 - 7 = 18$

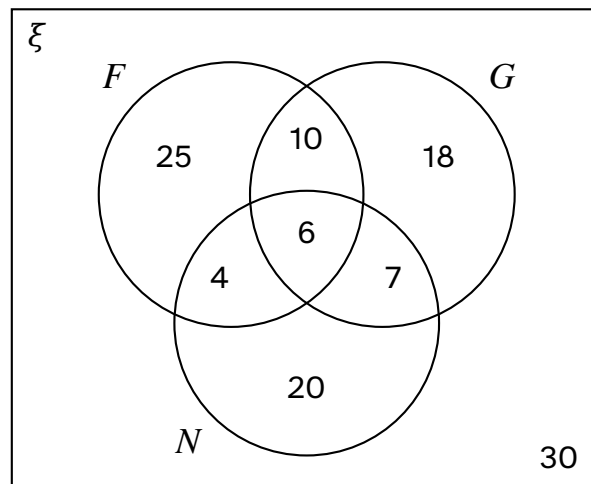
Outside all three: 30

Everyone else is in F only, N only or F and N only: $120 - 30 - 18 - 10 - 6 - 7 = 49$

F only + (F and N only) = $45 - 10 - 6 = 29$, so N only = $49 - 29 = 20$

F and N only: $37 - 20 - 6 - 7 = 4$

F only: $29 - 4 = 25$



Answer: See diagram

Tip: “13 belong to gym and netball” includes the 6 who belong to all three clubs. Check at the end that your eight numbers add up to 120.

Marks: 4 marks for a fully correct diagram. 3 marks for 6 or 7 regions correct. 2 marks for 4 or 5 regions correct. 1 mark for 6, 10 and 7 in the correct regions.

- (b) A student who belongs to exactly two of the clubs is chosen at random. Find the probability that one of these clubs is the gym.

Exactly two clubs: the three “two-only” regions, $10 + 4 + 7 = 21$ students.

The gym is one of them for F and G only (10) and G and N only (7): 17 students.

Answer: $\frac{17}{21}$

Tip: Do not include the 6 students in all three clubs: they belong to three clubs, not exactly two.

Marks: 1 mark for a fraction out of 21. 1 mark for $\frac{17}{21}$.

6 (7 marks)

There are 14 marbles in a jar.

r of the marbles are red and the rest are blue.

Bella takes at random two marbles from the jar without replacement.

The probability that Bella takes two marbles of the same colour is $\frac{46}{91}$

(a) Show that $r^2 - 14r + 45 = 0$

There are r red and $14 - r$ blue marbles. No replacement, so the second pick is out of 13.

Two red: $\frac{r}{14} \times \frac{r-1}{13} = \frac{r(r-1)}{182}$

Two blue: $\frac{14-r}{14} \times \frac{13-r}{13} = \frac{(14-r)(13-r)}{182}$

Same colour: $\frac{r(r-1) + (14-r)(13-r)}{182} = \frac{46}{91}$

Multiply both sides by 182: $r(r-1) + (14-r)(13-r) = 92$

Expand: $r^2 - r + 182 - 27r + r^2 = 92$

$2r^2 - 28r + 90 = 0$

Divide by 2: $r^2 - 14r + 45 = 0$

Answer: $r^2 - 14r + 45 = 0$ shown

Tip: "Same colour" means two red **or** two blue, so there are two products to add.

Marks: 1 mark for $\frac{r}{14} \times \frac{r-1}{13}$ or $\frac{14-r}{14} \times \frac{13-r}{13}$. 1 mark for adding both. 1 mark for setting equal to $\frac{46}{91}$ and clearing fractions. 1 mark for fully correct working to the given equation.

(b) There are more red marbles than blue marbles in the jar.

Work out the probability that the first marble Bella takes is blue and the second marble is red.

Solve $r^2 - 14r + 45 = 0$ by factorising: $(r-5)(r-9) = 0$

So $r = 5$ or $r = 9$.

There are more red marbles than blue, so $r = 9$ (and 5 blue).

Blue, then red: $\frac{5}{14} \times \frac{9}{13} = \frac{45}{182}$

Answer: $\frac{45}{182}$

Tip: Use the extra information to choose the right solution: $r = 5$ would give fewer red marbles than blue.

Marks: 1 mark for $(r - 5)(r - 9)$ or $r = 9$. 1 mark for $\frac{5}{14} \times \frac{9}{13}$. 1 mark for $\frac{45}{182}$.
