

Composite Functions

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

$$f(x) = \frac{x+7}{2} \quad \text{and} \quad g(x) = x^2 + 2$$

(a) Find $f(9)$

$$\begin{aligned} f(9) &= \frac{9+7}{2} \\ &= \frac{16}{2} = 8 \end{aligned}$$

Answer: 8

(b) Find $fg(3)$

$fg(3)$: work out $g(3)$ first.

$$g(3) = 3^2 + 2 = 11$$

Now put 11 into f :

$$f(11) = \frac{11+7}{2} = \frac{18}{2} = 9$$

Answer: 9

Tip: $fg(3)$ means do g first, then f . Working out $f(3)$ first gives $gf(3)$, which is a different number.

Marks: 1 mark for $g(3) = 11$. 1 mark for 9.

2 (8 marks)

$$f(x) = x^2 - 2 \quad g(x) = 2x + 1$$

(a) Find $fg(3)$

$fg(3)$: work out $g(3)$ first.

$$g(3) = 2 \times 3 + 1 = 7$$

Now put 7 into f :

$$f(7) = 7^2 - 2 = 47$$

Answer: 47

Tip: After finding $g(3) = 7$, put 7 into f . Don't use 3 again.

Marks: 1 mark for $g(3) = 7$. 1 mark for 47.

(b) Show that $gf(x) = 2x^2 - 3$

$$gf(x) = g(x^2 - 2)$$

g doubles its input, then adds 1:

$$2(x^2 - 2) + 1$$

$$= 2x^2 - 4 + 1$$

$$= 2x^2 - 3$$

Answer: Shown

Marks: 1 mark for $2(x^2 - 2) + 1$. 1 mark for a fully correct method.

(c) Solve $fg(x) = gf(x)$

$$fg(x) = f(2x + 1) = (2x + 1)^2 - 2$$

$$= 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1$$

$$\text{Set } fg(x) = gf(x): 4x^2 + 4x - 1 = 2x^2 - 3$$

$$\text{Rearrange: } 2x^2 + 4x + 2 = 0$$

$$\text{Divide by 2: } x^2 + 2x + 1 = 0$$

$$\text{Factorise: } (x + 1)^2 = 0$$

So $x = -1$ is the only solution.

Answer: $x = -1$

Tip: $(2x + 1)^2 = 4x^2 + 4x + 1$. Don't forget the middle term $4x$.

Marks: 1 mark for $(2x + 1)^2 - 2$. 1 mark for setting $fg(x) = gf(x)$. 1 mark for $2x^2 + 4x + 2 = 0$. 1 mark for $x = -1$.

3 (2 marks)

$$f(x) = \frac{1}{x-1} \text{ and } g(x) = 3x + 2$$

Find $fg(x)$ and simplify your answer.

$$fg(x) = f(3x + 2)$$

Replace x in $\frac{1}{x-1}$ with $(3x + 2)$:

$$\begin{aligned} & \frac{1}{3x + 2 - 1} \\ &= \frac{1}{3x + 1} \end{aligned}$$

Answer: $\frac{1}{3x + 1}$

Marks: 1 mark for $\frac{1}{3x+2-1}$. 1 mark for $\frac{1}{3x+1}$.

4 (4 marks)

$$f(x) = 2x - 1$$

(a) Find $ff(x)$

$ff(x)$ means apply f twice.

$$ff(x) = f(2x - 1)$$

$$= 2(2x - 1) - 1$$

$$= 4x - 2 - 1 = 4x - 3$$

Answer: $4x - 3$

Tip: $ff(x)$ is not $(f(x))^2$. You put $f(x)$ back into f .

Marks: 1 mark for $2(2x - 1) - 1$. 1 mark for $4x - 3$.

(b) Solve $ff(x) = x$

$$4x - 3 = x$$

Subtract x from both sides: $3x - 3 = 0$

$$3x = 3$$

$$x = 1$$

Answer: $x = 1$

Marks: 1 mark for $4x - 3 = x$. 1 mark for $x = 1$.

5 (2 marks)

$f(x) = x^2 + k$ where k is a constant.

$g(x) = 3x - 1$

Given that $gf(3) = 44$

work out the value of k .

$$f(3) = 3^2 + k = 9 + k$$

$$gf(3) = g(9 + k) = 3(9 + k) - 1$$

$$= 27 + 3k - 1 = 26 + 3k$$

$$\text{So } 26 + 3k = 44$$

$$3k = 18$$

$$k = 6$$

Answer: $k = 6$

Tip: Put $f(3) = 9 + k$ into g as a whole, with brackets: $3(9 + k) - 1$, not $3 \times 9 + k - 1$.

Marks: 1 mark for $3(9 + k) - 1 = 44$. 1 mark for $k = 6$.

6 (4 marks)

The functions f and g are such that

$$f(x) = x^2 \quad \text{and} \quad g(x) = x + 3$$

Solve $fg(x) = 4f(x)$

$$fg(x) = f(x + 3) = (x + 3)^2$$

$$4f(x) = 4x^2$$

$$\text{So } (x + 3)^2 = 4x^2$$

$$\text{Expand: } x^2 + 6x + 9 = 4x^2$$

$$\text{Make one side 0: } 3x^2 - 6x - 9 = 0$$

$$\text{Divide by 3: } x^2 - 2x - 3 = 0$$

$$\text{Factorise: } (x - 3)(x + 1) = 0$$

$$x = 3 \text{ or } x = -1$$

Answer: $x = 3$ or $x = -1$

Tip: $4f(x)$ means $4 \times f(x) = 4x^2$. It is not $f(4x) = 16x^2$.

Marks: 1 mark for $(x + 3)^2$. 1 mark for $(x + 3)^2 = 4x^2$. 1 mark for $x^2 - 2x - 3 = 0$ (or equivalent). 1 mark for both answers.

7 (4 marks)

$g(x) = x^2$ $h(x) = x + k$ where k is a constant.

Given that $gh(x) - hg(x) = 6x + 6$ for all values of x ,
work out the value of k .

You must show all your working.

$$gh(x) = g(x + k) = (x + k)^2 = x^2 + 2kx + k^2$$

$$hg(x) = h(x^2) = x^2 + k$$

$$gh(x) - hg(x) = x^2 + 2kx + k^2 - x^2 - k$$

$$= 2kx + k^2 - k$$

Compare with $6x + 6$.

The x terms: $2k = 6$, so $k = 3$

Check the numbers: $k^2 - k = 9 - 3 = 6$, which matches.

Answer: $k = 3$

Tip: $hg(x)$ means put x^2 into h , which gives $x^2 + k$. Take care with the minus sign when you subtract it: $-(x^2 + k) = -x^2 - k$.

Marks: 1 mark for $(x + k)^2$ expanded. 1 mark for $x^2 + k$. 1 mark for $2kx + k^2 - k$. 1 mark for $k = 3$.
