

Completing the Square

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Write $x^2 + 6x + 1$ in the form $(x + a)^2 + b$

Halve the coefficient of x : $6 \div 2 = 3$

$$(x + 3)^2 = x^2 + 6x + 9$$

$$\text{So } x^2 + 6x = (x + 3)^2 - 9$$

$$x^2 + 6x + 1 = (x + 3)^2 - 9 + 1$$

$$= (x + 3)^2 - 8$$

Answer: $(x + 3)^2 - 8$

Tip: The number in the bracket is **half** the coefficient of x . $(x + 6)^2$ is a common wrong start.

Marks: 1 mark for $(x + 3)^2$. 1 mark for the correct answer.

2 (4 marks)

(a) Write $x^2 + 12x + 5$ in the form $(x + a)^2 + b$

Halve 12 to get 6.

$$(x + 6)^2 = x^2 + 12x + 36$$

$$x^2 + 12x + 5 = (x + 6)^2 - 36 + 5$$

$$= (x + 6)^2 - 31$$

Answer: $(x + 6)^2 - 31$

Marks: 1 mark for $(x + 6)^2$. 1 mark for the correct answer.

(b) Write $x^2 - 3x - 2$ in the form $(x + a)^2 + b$

Halve -3 to get $-\frac{3}{2}$.

$$\left(x - \frac{3}{2}\right)^2 = x^2 - 3x + \frac{9}{4}$$

$$x^2 - 3x - 2 = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 2$$

$$-\frac{9}{4} - \frac{8}{4} = -\frac{17}{4}$$

Answer: $\left(x - \frac{3}{2}\right)^2 - \frac{17}{4}$

Marks: 1 mark for $\left(x - \frac{3}{2}\right)^2$. 1 mark for the correct answer.

3 (3 marks)

Solve $x^2 - 6x - 11 = 0$ by completing the square.

Give your answers in the form $p \pm q\sqrt{5}$

Complete the square: halve -6 to get -3 .

$$x^2 - 6x - 11 = (x - 3)^2 - 9 - 11 = (x - 3)^2 - 20$$

$$\text{So } (x - 3)^2 - 20 = 0$$

$$\text{Add 20 to both sides: } (x - 3)^2 = 20$$

$$\text{Square root both sides (remember } \pm \text{): } x - 3 = \pm\sqrt{20}$$

$$\text{Add 3: } x = 3 \pm \sqrt{20}$$

Simplify the surd: $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

Answer: $x = 3 \pm 2\sqrt{5}$

Marks: 1 mark for $(x - 3)^2 - 20$. 1 mark for $x - 3 = \pm\sqrt{20}$. 1 mark for the correct answer.

4 (3 marks)

Ella says,

“There is a value of x for which $x^2 - 8x + 22 = 5$ ”

By completing the square, show that Ella is wrong.

Complete the square: halve -8 to get -4 .

$$x^2 - 8x + 22 = (x - 4)^2 - 16 + 22$$

$$= (x - 4)^2 + 6$$

$(x - 4)^2 \geq 0$ for all x , because a square is never negative.

$$\text{So } x^2 - 8x + 22 = (x - 4)^2 + 6 \geq 6$$

The expression can never be less than 6, so it can never equal 5.

So Ella is wrong.

Answer: $(x - 4)^2 + 6 \geq 6$, so it can never equal 5

Tip: You must finish with a reason and a conclusion, e.g. “ $(x - 4)^2 \geq 0$, so the expression is at least 6, so it cannot be 5.”

Marks: 1 mark for $(x - 4)^2$. 1 mark for $(x - 4)^2 + 6$. 1 mark for a conclusion that uses $(x - 4)^2 \geq 0$.

5 (2 marks)

$x^2 + ax + 19 \equiv (x + b)^2 - 6$ where b is a positive integer.

Find the value of a and the value of b .

Expand the right side: $(x + b)^2 - 6 = x^2 + 2bx + b^2 - 6$

Compare the number terms: $b^2 - 6 = 19$

$b^2 = 25$, and b is positive, so $b = 5$

Compare the x terms: $a = 2b$

$$a = 2 \times 5 = 10$$

Answer: $a = 10$, $b = 5$

Tip: Start with the number terms here: $b^2 - 6$ must equal 19.

Marks: 1 mark for $b = 5$. 1 mark for $a = 10$.

6 (3 marks)

A curve has equation $y = x^2 + px + q$ where p and q are integers.
The turning point of the curve is $(5, -21)$.

(a) Find the value of p and the value of q .

The turning point is $(5, -21)$.

So the equation can be written as $y = (x - 5)^2 - 21$

Expand: $(x - 5)^2 - 21 = x^2 - 10x + 25 - 21$

$$= x^2 - 10x + 4$$

Compare with $x^2 + px + q$: $p = -10$ and $q = 4$

Answer: $p = -10$, $q = 4$

Tip: A turning point at $(5, -21)$ comes from $(x - 5)^2 - 21$. The sign in the bracket is the opposite of the x -coordinate.

Marks: 1 mark for $(x - 5)^2 - 21$. 1 mark for both values.

(b) Write down the coordinates of the point where the curve crosses the y -axis.

The curve crosses the y -axis when $x = 0$.

$$y = 0^2 - 10 \times 0 + 4 = 4$$

So the point is $(0, 4)$.

Answer: $(0, 4)$

7 (3 marks)

Solve $x^2 + 5x + 2 = 0$ by completing the square.

Give your answers in the form $\frac{a \pm \sqrt{b}}{2}$ where a and b are integers.

Complete the square: halve 5 to get $\frac{5}{2}$.

$$x^2 + 5x + 2 = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + 2$$

$$-\frac{25}{4} + \frac{8}{4} = -\frac{17}{4}$$

$$\text{So } \left(x + \frac{5}{2}\right)^2 - \frac{17}{4} = 0$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{17}{4}$$

$$\text{Square root both sides: } x + \frac{5}{2} = \pm \frac{\sqrt{17}}{2}$$

$$\text{Subtract } \frac{5}{2}: x = -\frac{5}{2} \pm \frac{\sqrt{17}}{2}$$

$$\text{Write as a single fraction: } x = \frac{-5 \pm \sqrt{17}}{2}$$

Answer: $x = \frac{-5 \pm \sqrt{17}}{2}$

Marks: 1 mark for $\left(x + \frac{5}{2}\right)^2 - \frac{17}{4}$. 1 mark for $x + \frac{5}{2} = \pm \sqrt{\frac{17}{4}}$. 1 mark for the correct answer.
