

Completing the Square - Harder

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Write $2x^2 + 8x + 1$ in the form $2(x + p)^2 + q$

Take out the factor 2 from the x^2 and x terms:

$$2(x^2 + 4x) + 1$$

Complete the square inside the bracket: $x^2 + 4x = (x + 2)^2 - 4$

So we get $2[(x + 2)^2 - 4] + 1$

Multiply out the square bracket: $2(x + 2)^2 - 8 + 1$

$$= 2(x + 2)^2 - 7$$

Answer: $2(x + 2)^2 - 7$

Tip: The -4 is inside the bracket, so it gets multiplied by 2: $2 \times (-4) = -8$. A common wrong answer is $2(x + 2)^2 - 3$.

Marks: 1 mark for $2[(x + 2)^2 - 4]$. 1 mark for the correct answer.

2 (3 marks)

Write $4x^2 + 24x + 11$ in the form $a(x + b)^2 + c$ where a , b and c are integers.

Take out the factor 4 from the x^2 and x terms:

$$4(x^2 + 6x) + 11$$

Complete the square inside the bracket: $x^2 + 6x = (x + 3)^2 - 9$

So we get $4[(x + 3)^2 - 9] + 11$

Multiply out the square bracket: $4(x + 3)^2 - 36 + 11$
 $= 4(x + 3)^2 - 25$

Answer: $4(x + 3)^2 - 25$

Marks: 1 mark for $4(x^2 + 6x)$. 1 mark for $4(x + 3)^2$. 1 mark for the correct answer.

3 (3 marks)

Write $3x^2 - 12x + 7$ in the form $a(x - b)^2 - c$

Take out the factor 3 from the x^2 and x terms:

$$3(x^2 - 4x) + 7$$

Complete the square inside the bracket: $x^2 - 4x = (x - 2)^2 - 4$

So we get $3[(x - 2)^2 - 4] + 7$

Multiply out the square bracket: $3(x - 2)^2 - 12 + 7$

$$= 3(x - 2)^2 - 5$$

Answer: $3(x - 2)^2 - 5$

Marks: 1 mark for $3(x^2 - 4x)$. 1 mark for $3(x - 2)^2$. 1 mark for the correct answer.

4 (2 marks)

Write $7 - 6x - x^2$ in the form $a - (x + b)^2$

Take out -1 from the x^2 and x terms:

$$7 - 6x - x^2 = -(x^2 + 6x) + 7$$

Complete the square inside the bracket: $x^2 + 6x = (x + 3)^2 - 9$

So we get $-[(x + 3)^2 - 9] + 7$

Multiply out the square bracket: $-(x + 3)^2 + 9 + 7$

$$= 16 - (x + 3)^2$$

Answer: $16 - (x + 3)^2$

Tip: Take out -1 first, and watch the signs: $-6x - x^2 = -(x^2 + 6x)$, and $-(-9) = +9$.

Marks: 1 mark for $(x + 3)^2$. 1 mark for the correct answer.

5 (2 marks)

$$2x^2 + px + q \equiv 2(x - 3)^2 + 1$$

Find the values of p and q .

Expand the right side:

$$(x - 3)^2 = x^2 - 6x + 9$$

$$2(x^2 - 6x + 9) + 1 = 2x^2 - 12x + 18 + 1$$

$$= 2x^2 - 12x + 19$$

Compare with $2x^2 + px + q$:

$$p = -12 \text{ and } q = 19$$

Answer: $p = -12, q = 19$

Marks: 1 mark for $p = -12$. 1 mark for $q = 19$.

6 (4 marks)

Solve $3x^2 + 6x - 2 = 0$ by completing the square.

Give your answers in the form $a \pm \sqrt{b}$

$$\text{Take out the factor 3: } 3(x^2 + 2x) - 2 = 0$$

$$\text{Complete the square: } x^2 + 2x = (x + 1)^2 - 1$$

$$3[(x + 1)^2 - 1] - 2 = 0$$

$$3(x + 1)^2 - 3 - 2 = 0$$

$$3(x + 1)^2 - 5 = 0$$

Add 5 to both sides: $3(x+1)^2 = 5$

Divide both sides by 3: $(x+1)^2 = \frac{5}{3}$

Square root both sides: $x+1 = \pm\sqrt{\frac{5}{3}}$

Subtract 1: $x = -1 \pm \sqrt{\frac{5}{3}}$

Answer: $x = -1 \pm \sqrt{\frac{5}{3}}$

Tip: Divide by 3 **before** you square root, and remember $\pm$.

Marks: 1 mark for $3(x+1)^2$. 1 mark for $3(x+1)^2 = 5$. 1 mark for $(x+1)^2 = \frac{5}{3}$. 1 mark for the correct answer.

7 (4 marks)

(a) Write $5x^2 + 20x + 26$ in the form $a(x+b)^2 + c$ where a , b and c are integers.

Take out the factor 5 from the x^2 and x terms:

$$5(x^2 + 4x) + 26$$

Complete the square inside the bracket: $x^2 + 4x = (x+2)^2 - 4$

So we get $5[(x+2)^2 - 4] + 26$

Multiply out the square bracket: $5(x+2)^2 - 20 + 26$

$$= 5(x+2)^2 + 6$$

Answer: $5(x+2)^2 + 6$

Tip: The -4 inside the square bracket must be multiplied by 5, giving -20 . A common error is to write $5(x+2)^2 - 4 + 26$.

Marks: 1 mark for $5(x^2 + 4x)$. 1 mark for $5(x+2)^2$. 1 mark for the correct answer.

(b) Hence, explain why the graph of $y = 5x^2 + 20x + 26$ does not cross the x -axis.

$$y = 5(x+2)^2 + 6$$

$5(x+2)^2$ is never negative, so y is always at least 6.

So y is never 0, and the graph never meets the x -axis.

Answer: The minimum value of y is 6 (at $x = -2$), so y is never 0

Tip: Use your answer to part (a): the turning point is $(-2, 6)$, which is above the x -axis, and the curve is U-shaped.

8 (3 marks)

A curve has equation $y = 5 + 12x - 3x^2$

Use completing the square to show that the highest point on the curve is $(2, 17)$

You must show your working.

Rearrange: $y = -3x^2 + 12x + 5$

Take out the factor -3 from the x^2 and x terms: $y = -3(x^2 - 4x) + 5$

Complete the square: $x^2 - 4x = (x - 2)^2 - 4$

So $y = -3[(x - 2)^2 - 4] + 5$

$= -3(x - 2)^2 + 12 + 5$

$= 17 - 3(x - 2)^2$

$3(x - 2)^2 \geq 0$, so the largest value of y is 17.

This happens when $x - 2 = 0$, so when $x = 2$.

So the maximum point is $(2, 17)$.

Answer: $y = 17 - 3(x - 2)^2$, so the maximum point is $(2, 17)$

Tip: Take care with the negative factor: $-3 \times (-4) = +12$. In a “show that” question, every step must be shown.

Marks: 1 mark for $-3(x^2 - 4x)$. 1 mark for $17 - 3(x - 2)^2$ (or $-3(x - 2)^2 + 17$). 1 mark for fully correct working with the coordinates.
