

Circle Theorems

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

A, B, C and D are points on the circumference of a circle.

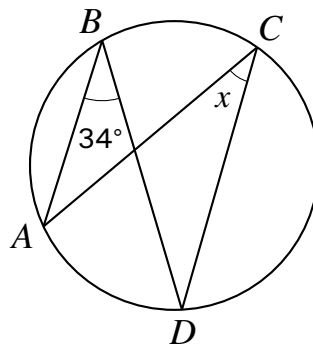


Diagram **NOT** accurately drawn

Find the size of angle x .

Give a reason for your answer.

Angles ABD and ACD both stand on the chord AD , on the same side.

$$x = 34^\circ$$

Reason: angles in the same segment are equal.

Answer: $x = 34^\circ$

Tip: Write the whole rule: “angles in the same segment are equal”. Just writing “same segment theorem” may not get the mark.

Marks: 1 mark for 34. 1 mark for the reason.

2 (2 marks)

A, B, C and D are points on a circle.

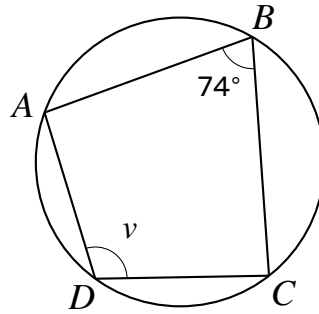


Diagram **NOT** accurately drawn

$ABCD$ is a cyclic quadrilateral.

Find the size of angle v .

Give a reason for your answer.

Angles v and 74° are opposite each other in the cyclic quadrilateral.

$$v = 180 - 74 = 106^\circ$$

Reason: opposite angles of a cyclic quadrilateral add up to 180° .

Answer: $v = 106^\circ$

Tip: Opposite angles of a cyclic quadrilateral add up to 180° ; they are not equal. Include the words “cyclic quadrilateral” in your reason.

Marks: 1 mark for 106. 1 mark for the reason.

3 (2 marks)

A , B and C are points on a circle.

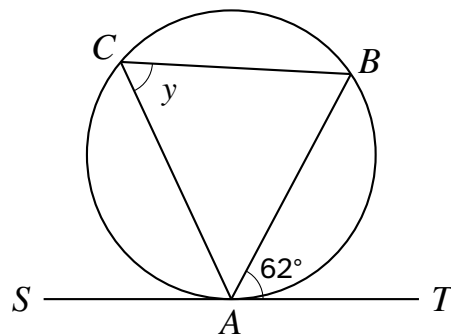


Diagram **NOT** accurately drawn

ST is the tangent to the circle at A .

Angle $BAT = 62^\circ$

Find the size of angle y .

Give a reason for your answer.

Angle BAT is between the tangent AT and the chord AB .

Angle y (angle ACB) is in the alternate segment.

So $y = 62^\circ$

Reason: alternate segment theorem. The angle between a tangent and a chord is equal to the angle in the alternate segment.

Answer: $y = 62^\circ$

Tip: Naming the theorem is not always enough. State what it says: the angle between a tangent and a chord equals the angle in the alternate segment.

Marks: 1 mark for 62. 1 mark for the reason.

4 (3 marks)

A , B and C are points on a circle, centre O .

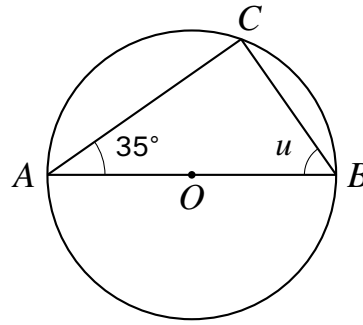


Diagram **NOT** accurately drawn

AB is a diameter of the circle.

Work out the size of angle u .

Give a reason for each stage of your working.

Angle $ACB = 90^\circ$

Reason: the angle in a semicircle is 90° .

$$u = 180 - 90 - 35 = 55^\circ$$

Reason: angles in a triangle add up to 180° .

Answer: $u = 55^\circ$

Marks: 1 mark for angle $ACB = 90^\circ$. 1 mark for 55. 1 mark for both reasons.

5 (3 marks)

A and B are points on a circle, centre O .

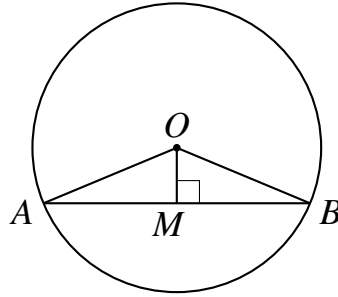


Diagram **NOT** accurately drawn

M is the point on the chord AB such that OM is perpendicular to AB .

$OA = 13$ cm and $AB = 24$ cm

Work out the length of OM .

The perpendicular from the centre to a chord bisects the chord.

So $AM = 24 \div 2 = 12$ cm

Triangle OAM has a right angle at M .

Pythagoras: $OM^2 = 13^2 - 12^2 = 169 - 144 = 25$

$OM = \sqrt{25} = 5$

Answer: 5 cm

Marks: 1 mark for $AM = 12$. 1 mark for $13^2 - 12^2$. 1 mark for 5.

6 (4 marks)

A , B and C are points on a circle, centre O .

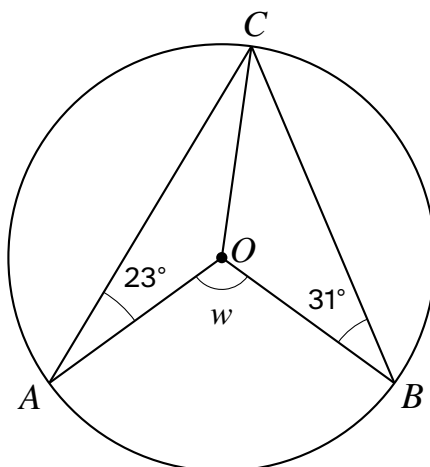


Diagram **NOT** accurately drawn

Angle $OAC = 23^\circ$ and angle $OBC = 31^\circ$

Work out the size of angle w .

You must give a reason for each stage of your working.

$OA = OC$ because they are both radii, so triangle OAC is isosceles.

Angle $OCA = 23^\circ$

Reason: base angles of an isosceles triangle are equal.

$OB = OC$ (radii), so in the same way angle $OCB = 31^\circ$

Angle $ACB = 23 + 31 = 54^\circ$

w is angle AOB , at the centre. Angle ACB is at the circumference. Both stand on the arc AB .

$w = 2 \times 54 = 108^\circ$

Reason: the angle at the centre is twice the angle at the circumference.

Answer: $w = 108^\circ$

Tip: Two radii make an isosceles triangle. Look for this whenever lines join the centre to two points on the circle.

Marks: 1 mark for angle $OCA = 23^\circ$ or angle $OCB = 31^\circ$. 1 mark for angle $ACB = 54^\circ$. 1 mark for 108. 1 mark for all the reasons.

7 (3 marks)

A and B are points on a circle, centre O .

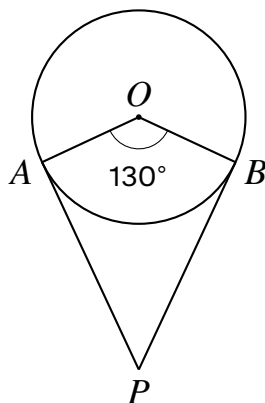


Diagram **NOT** accurately drawn

PA and PB are tangents to the circle from the point P .

Angle $AOB = 130^\circ$

Show that triangle PAB is isosceles and find angle PAB .

Give reasons for your answer.

$PA = PB$

Reason: tangents from an external point are equal in length.

So triangle PAB is isosceles.

Angle $OAP = \text{angle } OBP = 90^\circ$

Reason: a tangent is perpendicular to the radius.

In quadrilateral $OAPB$: angle $APB = 360 - 90 - 90 - 130 = 50^\circ$

Base angles of the isosceles triangle PAB are equal:

Angle $PAB = (180 - 50) \div 2 = 65^\circ$

Answer: Angle $PAB = 65^\circ$

Marks: 1 mark for $PA = PB$ with a reason. 1 mark for angle $APB = 50^\circ$. 1 mark for 65.

8 (4 marks)

A, B, C and D are points on a circle, centre O .

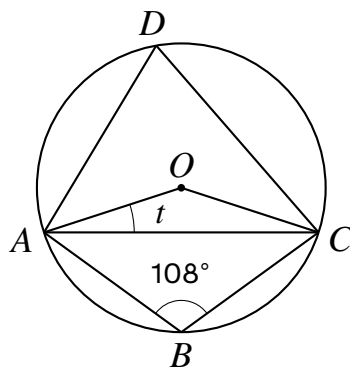


Diagram **NOT** accurately drawn

$ABCD$ is a cyclic quadrilateral. Angle $ABC = 108^\circ$

Work out the size of angle t .

Give a reason for each stage of your working.

$$\text{Angle } ADC = 180 - 108 = 72^\circ$$

Reason: opposite angles of a cyclic quadrilateral add up to 180° .

$$\text{Angle } AOC = 2 \times 72 = 144^\circ$$

Reason: the angle at the centre is twice the angle at the circumference.

$OA = OC$ (radii), so triangle AOC is isosceles.

$$t = (180 - 144) \div 2 = 18^\circ$$

Reason: base angles of an isosceles triangle are equal, and angles in a triangle add up to 180° .

Answer: $t = 18^\circ$

Tip: $OABC$ is not a cyclic quadrilateral, because O is not on the circle. Only use cyclic quadrilateral facts for $ABCD$.

Marks: 1 mark for angle $ADC = 72^\circ$. 1 mark for angle $AOC = 144^\circ$. 1 mark for 18. 1 mark for all the reasons.

9 (3 marks)

A, B, C and D are points on a circle.

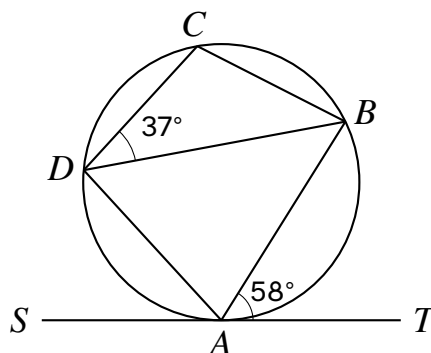


Diagram **NOT** accurately drawn

SAT is the tangent to the circle at A .

Angle $TAB = 58^\circ$ and angle $BDC = 37^\circ$

Work out the size of angle ADC .

Give a reason for each stage of your working.

Angle TAB is between the tangent AT and the chord AB .

Angle ADB stands on the chord AB in the alternate segment.

So angle $ADB = 58^\circ$

Reason: alternate segment theorem. The angle between a tangent and a chord is equal to the angle in the alternate segment.

Angle $ADC = \text{angle } ADB + \text{angle } BDC$

Angle $ADC = 58 + 37 = 95^\circ$

Answer: Angle $ADC = 95^\circ$

Tip: To use the alternate segment theorem, look for the angle on the **other side** of the chord AB from the tangent angle. Here that is angle ADB (or angle ACB).

Marks: 1 mark for angle $ADB = 58^\circ$. 1 mark for 95. 1 mark for the alternate segment theorem as the reason.

10 (3 marks)

A , B , C and D are points on a circle.

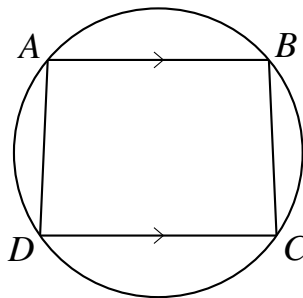


Diagram **NOT** accurately drawn

AB is parallel to DC .

Prove that angle ADC = angle BCD .

Let angle $ADC = x$.

Angle $DAB = 180^\circ - x$

Reason: co-interior angles add up to 180° (AB is parallel to DC).

$ABCD$ is a cyclic quadrilateral, so angle $BCD = 180^\circ - \text{angle } DAB$

Reason: opposite angles of a cyclic quadrilateral add up to 180° .

Angle $BCD = 180^\circ - (180^\circ - x) = x$

So angle ADC = angle BCD .

Tip: Use a letter such as x for the unknown angle, not a number: a worked example with one value is not a proof. Give a reason for every step, written out in full (e.g. “co-interior angles add up to 180° ”, not “C angles”).

Marks: 1 mark for angle $DAB = 180^\circ - \text{angle } ADC$ with the co-interior angles reason. 1 mark for angle $BCD = 180^\circ - \text{angle } DAB$ with the cyclic quadrilateral reason. 1 mark for a complete proof.