

Capture-Recapture

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Ria catches 30 beetles in her garden. She marks them and releases them.
A week later she catches 50 beetles. 6 of these beetles are marked.

(a) What fraction of the beetles in the second sample are marked?

6 out of the 50 beetles in the second sample are marked.

Answer: $\frac{6}{50} = \frac{3}{25}$

(b) Assume that the same fraction of all the beetles in the garden are marked.
Work out an estimate for the number of beetles in the garden.

Let N be the number of beetles in the garden.

30 of them are marked, so the fraction marked is $\frac{30}{N}$.

This should equal the fraction in the sample: $\frac{30}{N} = \frac{6}{50}$

$$N = \frac{30 \times 50}{6}$$

$$N = 250$$

Answer: 250

Marks: 1 mark for $\frac{30 \times 50}{6}$. 1 mark for 250.

2 (4 marks)

Alex wants to estimate the number of fish in a lake.
He catches 60 fish, marks them and puts them back.
Later he catches 40 fish. 8 of these are marked.

- (a) Work out an estimate for the number of fish in the lake.

Let N be the number of fish in the lake.

Fraction marked in the lake = fraction marked in the second sample:

$$\frac{60}{N} = \frac{8}{40}$$

$$N = \frac{60 \times 40}{8}$$

$$N = 300$$

Answer: 300

Tip: Use fractions (ratios), not differences. Wrong methods include $60 + 40 - 8$ and 60×8 .

Marks: 1 mark for $\frac{8}{40}$ (or $\frac{40}{8}$). 1 mark for $\frac{60 \times 40}{8}$. 1 mark for 300.

- (b) Write down one assumption you have made.

Any one of these:

The number of fish in the lake does not change (no fish are born, die, arrive or leave).

The marks do not come off.

The marked fish mix in with the others, so the second sample is random.

Answer: Population stays the same / marks not lost

Tip: An assumption is something you take to be true about the fish or the marks. "All the fish were caught" is not a valid assumption.

3 (3 marks)

A scientist marks 120 birds on an island and releases them.
A week later she catches 75 birds and finds that 18 are marked.
Work out an estimate for the number of birds on the island.

Let N be the number of birds on the island.

$$\frac{120}{N} = \frac{18}{75}$$

$$N = \frac{120 \times 75}{18}$$

$$N = 9000 \div 18 = 500$$

Answer: 500

Marks: 1 mark for $\frac{18}{75}$ (or $\frac{75}{18}$). 1 mark for $\frac{120 \times 75}{18}$. 1 mark for 500.

4 (2 marks)

An estimate of 400 frogs in a pond was made. 50 frogs were marked first.
In the second sample, 5 frogs were marked.
How many frogs were in the second sample?

Let n be the number of frogs in the second sample.

Fraction marked in the pond = fraction marked in the sample:

$$\frac{50}{400} = \frac{5}{n}$$

$$\frac{50}{400} = \frac{1}{8}, \text{ so } 5 \text{ is } \frac{1}{8} \text{ of } n.$$

$$n = 5 \times 8 = 40$$

Answer: 40

Marks: 1 mark for a correct equation, e.g. $\frac{50 \times n}{5} = 400$. 1 mark for 40.

5 (1 mark)

Explain why capture-recapture gives only an estimate.

It relies on the second sample being representative of the whole population.

A different sample could have a different number of marked animals.

So it could give a different result.

Answer: It is based on a sample. A different sample would probably contain a different number of marked animals, so it would give a different estimate.

6 (4 marks)

A ranger wants to estimate the number of deer in a forest.

In March, she puts a collar on 45 deer and releases them.

In April, she counts 80 deer. 12 of these deer have collars.

- (a) Work out an estimate for the number of deer in the forest.

Let N be the number of deer in the forest.

Fraction with collars in the forest = fraction with collars in the April count:

$$\frac{45}{N} = \frac{12}{80}$$

$$N = \frac{45 \times 80}{12}$$

$$N = 3600 \div 12 = 300$$

Answer: 300

Marks: 1 mark for $\frac{12}{80}$ (or $\frac{80}{12}$). 1 mark for $\frac{45 \times 80}{12}$. 1 mark for 300.

- (b) Deer with collars are easier to spot than deer without collars.
Explain what effect this could have on the estimate in part (a).

If collared deer are easier to spot, the ranger will count too many of them in April.

So the fraction with collars in her count is too big.

Dividing by a bigger fraction gives a smaller answer.

So the estimate will be too small (an underestimate).

Answer: The estimate would be too small (an underestimate), because too many collared deer are counted in April

Tip: Say which way the estimate changes (too big or too small) and why. “It will be less accurate” on its own does not get the mark.

7 (4 marks)

Lena and Kofi want to estimate the number of newts in a pond.

On Monday, they catch 60 newts, put a small mark on each one and release them.

On Tuesday, Lena catches 50 newts and finds that 6 of them are marked.

At the same time, Kofi catches 70 different newts and finds that 9 of them are marked.

- (a) Use all of these results to work out an estimate for the number of newts in the pond.

You must show all your working.

Put the two Tuesday samples together.

Newts caught on Tuesday: $50 + 70 = 120$

Marked newts caught on Tuesday: $6 + 9 = 15$

Let N be the number of newts in the pond.

$$\frac{60}{N} = \frac{15}{120} = \frac{1}{8}$$

$$N = 60 \times 8 = 480$$

Answer: 480

Tip: Add the samples first. Working out two separate estimates (500 and 466.7) and not combining them correctly is a common mistake.

Marks: 1 mark for 120 and 15. 1 mark for $\frac{60 \times 120}{15}$. 1 mark for 480.

- (b) Explain why the estimate in part (a) is likely to be more reliable than an estimate that uses only Lena’s sample.

Part (a) uses a sample of 120 newts.

Lena’s sample has only 50 newts.

A bigger sample is more likely to represent the whole population.

Answer: It uses a bigger sample (120 newts rather than 50)
