

Bearings with Trigonometry

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

From a campsite W , the bearing of a radio mast M is 206° .
Work out the bearing of W from M .

The bearing of W from M is in the opposite direction.

206° is more than 180° , so subtract 180° :

$$206 - 180 = 26$$

Bearings are written with three figures: 026°

Answer: 026°

Tip: Check with a sketch. M is South West of W , so W must be North East of M (between 000° and 090°). 386° is not a bearing.

2 (3 marks)

A ship sails 12 km from a harbour on a bearing of 050°

- (a) Work out how far east of the harbour the ship is.
Give your answer correct to 3 significant figures.

Draw a right-angled triangle: 12 km is the hypotenuse.

The angle at the harbour, between North and the ship's path, is 50° .

The distance east is opposite the 50° angle, so use sine:

$$\text{Distance east} = 12 \sin 50^\circ = 9.192 \dots$$

$$= 9.19 \text{ (3 s.f.)}$$

Answer: 9.19 km

Tip: The 50° is measured from North. So “east” is opposite the angle (sin), and “north” is next to it (cos).

Marks: 1 mark for $12 \sin 50^\circ$. 1 mark for 9.19.

- (b) Work out how far north of the harbour the ship is.
Give your answer correct to 3 significant figures.

The distance north is adjacent to the 50° angle, so use cosine:

$$\text{Distance north} = 12 \cos 50^\circ = 7.713 \dots$$

$$= 7.71 \text{ (3 s.f.)}$$

Answer: 7.71 km

3 (3 marks)

Town B is 25 km from town A on a bearing of 050°

Town C is 18 km from town A on a bearing of 140°

Work out the area of triangle ABC .

$$\text{Angle } BAC = 140 - 50 = 90^\circ$$

So triangle ABC is right-angled at A .

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 25 \times 18 = 225$$

Answer: 225 km²

Tip: Always find the angle between the two bearings first. Here it is a right angle, which makes the area easy.

Marks: 1 mark for angle $BAC = 90^\circ$. 1 mark for $\frac{1}{2} \times 25 \times 18$. 1 mark for 225.

4 (4 marks)

Port P is 20 km from port Q on a bearing of 300° from Q .

Port R is 15 km from Q on a bearing of 020°

Calculate the distance PR .

Sketch both ports with a North line at Q .

P is $360 - 300 = 60^\circ$ anticlockwise from North.

R is 20° clockwise from North.

So angle $PQR = 60 + 20 = 80^\circ$

Use the cosine rule:

$$PR^2 = 20^2 + 15^2 - 2 \times 20 \times 15 \times \cos 80^\circ$$

$$PR^2 = 625 - 104.1 \dots = 520.8 \dots$$

$$PR = 22.82 \dots$$

$$= 22.8 \text{ (3 s.f.)}$$

Answer: 22.8 km

Marks: 1 mark for angle $PQR = 80^\circ$. 1 mark for substituting into the cosine rule. 1 mark for $PR^2 = 520.8 \dots$ 1 mark for 22.8.

5 (4 marks)

Two coastguard stations, A and B , are on a straight coastline.

B is 15 km due East of A .

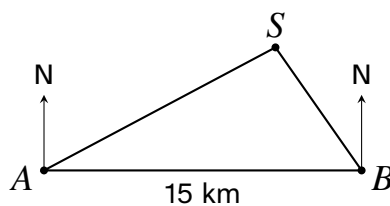


Diagram **NOT** accurately drawn

A ship S is on a bearing of 062° from A and on a bearing of 325° from B .

Calculate the distance AS .

Give your answer correct to 3 significant figures.

B is due East of A (bearing 090°).

$$\text{Angle } SAB = 90 - 62 = 28^\circ$$

A is due West of B (bearing 270°).

$$\text{Angle } SBA = 325 - 270 = 55^\circ$$

$$\text{Angles in a triangle: angle } ASB = 180 - 28 - 55 = 97^\circ$$

Use the sine rule. AS is opposite angle B , and 15 km is opposite angle S :

$$\frac{AS}{\sin 55^\circ} = \frac{15}{\sin 97^\circ}$$

$$AS = \frac{15 \sin 55^\circ}{\sin 97^\circ} = 12.37 \dots$$

$$= 12.4 \text{ (3 s.f.)}$$

Answer: 12.4 km

Tip: Pair each side with the angle opposite it. AS is opposite angle B (55°), not angle A .

Marks: 1 mark for 28° and 55° . 1 mark for 97° . 1 mark for a correct sine rule. 1 mark for 12.4.

6 (5 marks)

A lighthouse L is 9 km due North of a port P .

A ship S is 14 km from P on a bearing of 115°

Work out the bearing of S from L .

Give your answer correct to the nearest degree.

Sketch P with L directly above it (North) and S to the South East.

Angle $LPS = 115^\circ$ (the bearing of S from P , measured from North).

Find LS with the cosine rule:

$$LS^2 = 9^2 + 14^2 - 2 \times 9 \times 14 \times \cos 115^\circ = 383.4 \dots$$

$$LS = 19.58 \dots \text{ km}$$

Find angle PLS with the sine rule:

$$\frac{\sin PLS}{14} = \frac{\sin 115^\circ}{19.58 \dots}$$

$$\sin PLS = 0.6479 \dots, \text{ so angle } PLS = 40.38 \dots^\circ$$

From L , P is due South (bearing 180°), and S is $40.38 \dots^\circ$ round from South towards East.

Bearing of S from $L = 180 - 40.38 \dots = 139.6 \dots$

$= 140^\circ$ (nearest degree)

Answer: 140°

Tip: Angle PLS is not the bearing. Draw a North line at L : the bearing is measured clockwise from North, so it is $180^\circ - 40.4^\circ$.

Marks: 1 mark for using the cosine rule with 115° . 1 mark for $LS = 19.6 \dots$ 1 mark for a correct method for angle PLS (sine rule or cosine rule). 1 mark for 40.4° . 1 mark for 140 .

7 (5 marks)

A , B and C are three buoys.

B is 12 km due East of A .

C is 7 km from A and is North of the line AB .

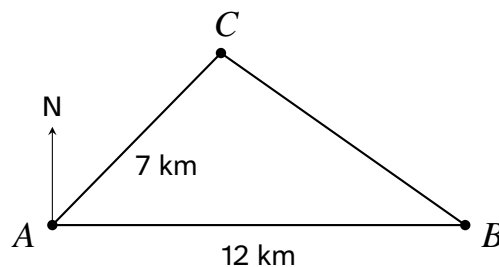


Diagram **NOT** accurately drawn

The area of triangle ABC is 30 km^2

Angle BAC is acute.

Find the bearing of C from A .

Give your answer correct to 1 decimal place.

You must show all your working.

$$\text{Area} = \frac{1}{2} \times AB \times AC \times \sin BAC$$

$$30 = \frac{1}{2} \times 12 \times 7 \times \sin BAC$$

$$30 = 42 \sin BAC$$

$$\sin BAC = \frac{30}{42} = 0.7142 \dots$$

Angle $BAC = \sin^{-1}(0.7142 \dots) = 45.58 \dots^\circ$ (acute)

From A , B is due East, a bearing of 090° .

C is $45.58 \dots^\circ$ anticlockwise from East (towards North).

Bearing of C from $A = 90 - 45.58 \dots = 44.41 \dots$

$= 044.4^\circ$ (1 d.p.)

Answer: 044.4°

Tip: Use Area $= \frac{1}{2}ab \sin C$ with the two sides that meet at A . Then turn angle BAC into a bearing using the North line at A .

Marks: 1 mark for $\frac{1}{2} \times 12 \times 7 \times \sin BAC = 30$. 1 mark for $\sin BAC = \frac{30}{42}$. 1 mark for 45.6° . 1 mark for $90 - 45.58 \dots$. 1 mark for 044.4 .
