

Area of a Triangle Using Sine

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

ABC is a triangle.

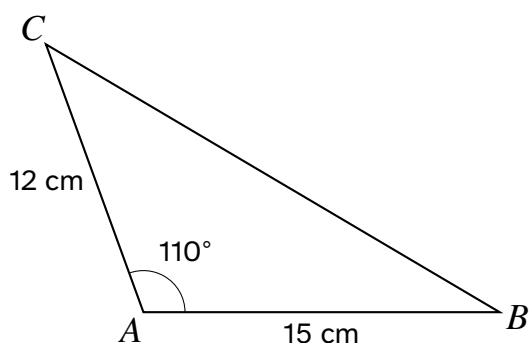


Diagram **NOT** accurately drawn

Work out the area of triangle ABC .

Give your answer correct to 3 significant figures.

The 110° angle is between the 12 cm and 15 cm sides.

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area} = \frac{1}{2} \times 12 \times 15 \times \sin 110^\circ = 84.57 \dots$$

Answer: 84.6 cm²

Tip: The triangle is not right-angled, so $\frac{1}{2} \times 12 \times 15$ is wrong. You need the $\sin 110^\circ$.

Marks: 1 mark for $\frac{1}{2} \times 12 \times 15 \times \sin 110^\circ$. 1 mark for 84.6.

2 (2 marks)

Here is an equilateral triangle.

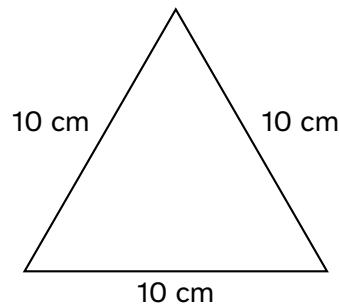


Diagram **NOT** accurately drawn

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

Work out the area of the triangle.

Give your answer in the form $a\sqrt{3}$

All the angles of an equilateral triangle are 60° .

$$\text{Area} = \frac{1}{2} \times 10 \times 10 \times \sin 60^\circ$$

$$= 50 \times \frac{\sqrt{3}}{2}$$

$$= 25\sqrt{3}$$

Answer: $25\sqrt{3} \text{ cm}^2$

Marks: 1 mark for $\frac{1}{2} \times 10 \times 10 \times \sin 60^\circ$. 1 mark for $25\sqrt{3}$.

3 (3 marks)

A triangle has area $18\sqrt{3} \text{ cm}^2$.

Two of its sides have lengths 8 cm and x cm, and the angle between them is 60° .

Find the value of x .

$$\text{Area} = \frac{1}{2} \times 8 \times x \times \sin 60^\circ$$

$$\text{Use the exact value } \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{So } \frac{1}{2} \times 8 \times x \times \frac{\sqrt{3}}{2} = 18\sqrt{3}$$

Simplify the left side: $\frac{1}{2} \times 8 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

$$2\sqrt{3}x = 18\sqrt{3}$$

Divide both sides by $2\sqrt{3}$: $x = 9$

Answer: $x = 9$

Tip: You are expected to know the exact value $\sin 60^\circ = \frac{\sqrt{3}}{2}$. Then the $\sqrt{3}$ on each side cancels.

Marks: 1 mark for $\frac{1}{2} \times 8 \times x \times \sin 60^\circ = 18\sqrt{3}$. 1 mark for $2\sqrt{3}x = 18\sqrt{3}$. 1 mark for 9.

4 (5 marks)

PQR is a triangle with $PQ = 7$ cm, $QR = 9$ cm and $PR = 12$ cm.

Work out the area of the triangle.

Give your answer correct to 3 significant figures.

We know three sides but no angles.

So first use the cosine rule to find an angle, e.g. angle Q .

Angle Q is opposite $PR = 12$.

$$\cos Q = \frac{7^2 + 9^2 - 12^2}{2 \times 7 \times 9} = \frac{-14}{126}$$

$$Q = \cos^{-1}\left(\frac{-14}{126}\right) = 96.37 \dots^\circ$$

Angle Q is between sides $PQ = 7$ and $QR = 9$.

$$\text{Area} = \frac{1}{2} \times 7 \times 9 \times \sin 96.37 \dots^\circ = 31.30 \dots$$

Answer: 31.3 cm²

Tip: You cannot put all three sides into $\frac{1}{2}ab \sin C$. Find an angle first, then use the two sides next to it.

Marks: 1 mark for using the cosine rule. 1 mark for $\cos Q = -\frac{14}{126}$. 1 mark for 96.4° . 1 mark for $\frac{1}{2} \times 7 \times 9 \times \sin Q$. 1 mark for 31.3.

5 (4 marks)

PQR is a triangle.

$PQ = 10$ cm and angle $QPR = 42^\circ$

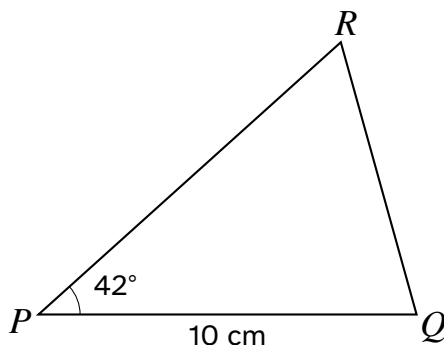


Diagram **NOT** accurately drawn

The area of triangle PQR is 36 cm^2 .

Calculate the length of QR .

Give your answer correct to 3 significant figures.

First use the area to find PR .

$$\frac{1}{2} \times 10 \times PR \times \sin 42^\circ = 36$$

$$PR = \frac{72}{10 \sin 42^\circ} = 10.76 \dots \text{ cm}$$

Now use the cosine rule in triangle PQR . QR is opposite the 42° angle.

$$QR^2 = 10^2 + (10.76 \dots)^2 - 2 \times 10 \times 10.76 \dots \times \cos 42^\circ$$

$$QR^2 = 215.78 \dots - 159.93 \dots = 55.85 \dots$$

$$QR = \sqrt{55.85 \dots} = 7.473 \dots$$

$$= 7.47 \text{ cm (3 s.f.)}$$

Answer: 7.47 cm

Tip: Keep the full calculator value of PR for the cosine rule. Rounding PR to 10.8 first gives 7.49, which is not accurate enough.

Marks: 1 mark for $\frac{1}{2} \times 10 \times PR \times \sin 42^\circ = 36$. 1 mark for $PR = 10.76 \dots$ 1 mark for the cosine rule with 42° . 1 mark for 7.47.

6 (5 marks)

Here is triangle ABC .

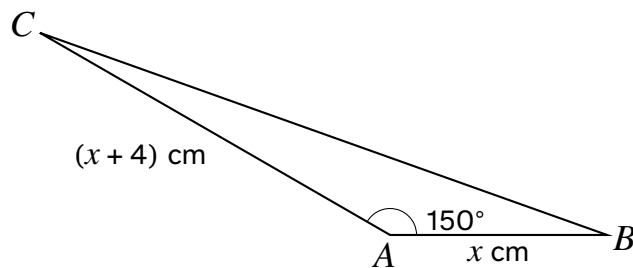


Diagram **NOT** accurately drawn

The area of triangle ABC is 10 cm^2 .

Calculate the value of x .

Give your answer correct to 3 significant figures.

The 150° angle is between the sides x and $(x + 4)$.

$$\frac{1}{2} \times x \times (x + 4) \times \sin 150^\circ = 10$$

$$\sin 150^\circ = 0.5, \text{ so } \frac{1}{4}x(x + 4) = 10$$

$$x(x + 4) = 40$$

$$\text{Expand: } x^2 + 4x = 40$$

$$x^2 + 4x - 40 = 0$$

Use the quadratic formula with $a = 1$, $b = 4$, $c = -40$:

$$x = \frac{-4 \pm \sqrt{16 + 160}}{2} = \frac{-4 \pm \sqrt{176}}{2}$$

$$x = 4.633 \dots \text{ or } x = -8.63 \dots$$

A length cannot be negative, so $x = 4.633 \dots$

$$x = 4.63 \text{ (3 s.f.)}$$

Answer: $x = 4.63$

Tip: $\sin 150^\circ = \sin 30^\circ = 0.5$. The angle is obtuse but its sine is still positive. Give only the positive solution, as x is a length.

Marks: 1 mark for $\frac{1}{2}x(x+4)\sin 150^\circ = 10$. 1 mark for $x(x+4) = 40$. 1 mark for $x^2 + 4x - 40 = 0$. 1 mark for using the quadratic formula (or completing the square). 1 mark for 4.63.

7 (5 marks)

$ABCDEFGH$ is a regular octagon.

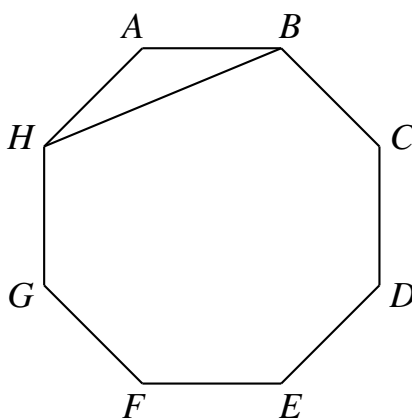


Diagram **NOT** accurately drawn

The area of triangle ABH is 20 cm^2 .

Calculate the length of HB .

Give your answer correct to 3 significant figures.

You must show all your working.

Exterior angle of a regular octagon $= 360 \div 8 = 45^\circ$

Interior angle $= 180 - 45 = 135^\circ$, so angle $HAB = 135^\circ$

Call the side length s . Then $AB = AH = s$.

Area of ABH : $\frac{1}{2} \times s \times s \times \sin 135^\circ = 20$

$$s^2 = \frac{40}{\sin 135^\circ} = 56.56 \dots$$

Now use the cosine rule in triangle ABH :

$$HB^2 = s^2 + s^2 - 2 \times s \times s \times \cos 135^\circ$$

$$HB^2 = 56.56 \dots + 56.56 \dots + 80 = 193.1 \dots$$

$$HB = \sqrt{193.1 \dots} = 13.89 \dots$$

Answer: 13.9 cm

Tip: 45° is the exterior angle. The angle inside the triangle at A is the interior angle, 135° .

Marks: 1 mark for 135° . 1 mark for $\frac{1}{2}s^2 \sin 135^\circ = 20$. 1 mark for $s^2 = 56.6$ or $s = 7.52$. 1 mark for the cosine rule for HB . 1 mark for 13.9.
