

Area and Perimeter of Compound Shapes with Circles

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

The diagram shows two circles with the same centre.
The larger circle has diameter 14 cm.

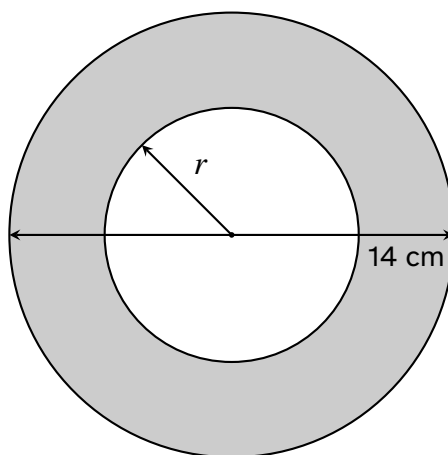


Diagram **NOT** accurately drawn

The area of the shaded ring is $33\pi \text{ cm}^2$.
Work out the radius, r , of the smaller circle.

The larger circle has diameter 14 cm, so its radius is 7 cm.

$$\text{Area of larger circle} = \pi \times 7^2 = 49\pi$$

$$\text{Area of smaller circle} = 49\pi - 33\pi = 16\pi$$

$$\pi r^2 = 16\pi, \text{ so } r^2 = 16$$

$$r = 4$$

Answer: 4 cm

Tip: Halve the diameter first: the larger circle has radius 7 cm, so its area is 49π , not 196π .

Marks: 1 mark for 49π . 1 mark for $49\pi - 33\pi$ (or 16π). 1 mark for 4.

2 (3 marks)

A circular pond has an area of 20 m^2 .

Work out the circumference of the pond.

Give your answer correct to 3 significant figures.

First find the radius from the area.

$$\pi r^2 = 20$$

$$r^2 = \frac{20}{\pi} = 6.366 \dots$$

$$r = \sqrt{6.366 \dots} = 2.523 \dots \text{ m}$$

$$\text{Circumference} = 2\pi r = 2 \times \pi \times 2.523 \dots$$

$$= 15.85 \dots$$

$$= 15.9 \text{ m (3 s.f.)}$$

Answer: 15.9 m

Tip: Don't forget to square root: $\frac{20}{\pi}$ is r^2 , not r . Keep the full calculator value of r until the end.

Marks: 1 mark for $r^2 = \frac{20}{\pi}$. 1 mark for $r = 2.52 \dots$. 1 mark for 15.9.

3 (3 marks)

A trundle wheel is used to measure the length of a path.

The wheel has a radius of 25 cm.

The wheel makes 58 complete turns along the path.

Work out the length of the path.

Give your answer in metres, correct to 3 significant figures.

In one turn, the wheel travels one circumference.

$$\text{Circumference} = 2\pi r = 2 \times \pi \times 25 = 157.07 \dots \text{ cm}$$

$$\text{Length of path} = 58 \times 157.07 \dots = 9110.6 \dots \text{ cm}$$

$$\text{Change to metres: } 9110.6 \dots \div 100 = 91.10 \dots \text{ m}$$

$$= 91.1 \text{ m (3 s.f.)}$$

Answer: 91.1 m

Tip: The radius is 25 cm, so the circumference is $\pi \times 50$, not $\pi \times 25$. Change centimetres to metres at the end.

Marks: 1 mark for $2 \times \pi \times 25$ (or $\pi \times 50$). 1 mark for $58 \times$ your circumference. 1 mark for 91.1.

4 (3 marks)

The perimeter of a semicircle is 36 cm.

Work out the radius of the semicircle. Give your answer correct to 3 significant figures.

Let the radius be r .

The perimeter is the curved arc plus the straight diameter.

$$\text{Arc} = \frac{1}{2} \times 2\pi r = \pi r$$

$$\text{Diameter} = 2r$$

$$\text{So } \pi r + 2r = 36$$

$$\text{Factorise: } r(\pi + 2) = 36$$

$$r = \frac{36}{\pi + 2} = 7.001 \dots$$

$$r = 7.00 \text{ (3 s.f.)}$$

Answer: 7.00 cm

Tip: The perimeter of a semicircle includes the straight edge (the diameter, $2r$), not just the curved part.

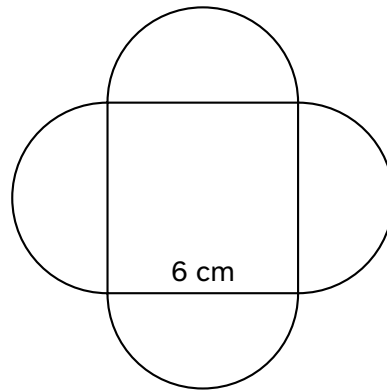
Marks: 1 mark for $\pi r + 2r$. 1 mark for $r = \frac{36}{\pi+2}$. 1 mark for 7.00.

5 (4 marks)

The shape is made from a square and four semicircles.

The square has sides of length 6 cm.

Each side of the square is the diameter of one of the semicircles.



*Diagram **NOT** accurately drawn*

- (a) Work out the perimeter of the shape.
Give your answer correct to 3 significant figures.

The perimeter is made of the four curved edges only (the sides of the square are inside the shape).

Each semicircle has diameter 6 cm.

$$\text{One arc} = \frac{1}{2} \times \pi \times 6 = 3\pi$$

$$\text{Perimeter} = 4 \times 3\pi = 12\pi = 37.69 \dots$$

$$= 37.7 \text{ cm (3 s.f.)}$$

Answer: 37.7 cm

Tip: Do not add the sides of the square: they are inside the shape, not on its edge.

Marks: 1 mark for $\frac{1}{2} \times \pi \times 6$ (or $4 \times 3\pi$). 1 mark for 37.7.

(b) Work out the area of the shape.

Give your answer correct to 3 significant figures.

$$\text{Area of square} = 6 \times 6 = 36 \text{ cm}^2$$

Each semicircle has radius 3 cm.

$$\text{Four semicircles make two whole circles: } 2 \times \pi \times 3^2 = 18\pi = 56.54 \dots \text{ cm}^2$$

$$\text{Total area} = 36 + 56.54 \dots = 92.54 \dots$$

$$= 92.5 \text{ cm}^2 \text{ (3 s.f.)}$$

Answer: 92.5 cm²

Tip: Use the radius (3 cm), not the diameter, in πr^2 .

Marks: 1 mark for $36 + 4 \times \frac{1}{2} \times \pi \times 3^2$. 1 mark for 92.5.

6 (3 marks)

The diagram shows a square $ABCD$ and a circle.

The circle touches all four sides of $ABCD$.

$PQRS$ is a square with its vertices P , Q , R and S on the circle.

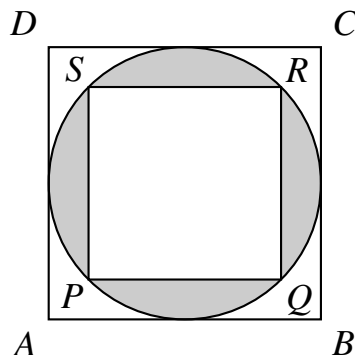


Diagram **NOT** accurately drawn

Show that $\frac{\text{area of shaded region}}{\text{area of square } ABCD} = \frac{\pi - 2}{4}$

Let the radius of the circle be r .

The circle touches all four sides of $ABCD$, so each side of $ABCD$ is $2r$.

$$\text{Area of } ABCD = (2r)^2 = 4r^2$$

PR is a diagonal of $PQRS$ and a diameter of the circle, so $PR = 2r$.

Let the side of $PQRS$ be x . By Pythagoras, $x^2 + x^2 = (2r)^2$, so $x^2 = 2r^2$.

$$\text{Area of } PQRS = 2r^2$$

$$\text{Shaded region} = \pi r^2 - 2r^2$$

$$\frac{\pi r^2 - 2r^2}{4r^2} = \frac{\pi - 2}{4} \text{ as required.}$$

Answer: Shown

Tip: You can use any letter (or a number) for the radius. The diagonal of the small square is a diameter of the circle, not a side of the big square.

Marks: 1 mark for area of $ABCD = 4r^2$ (or a value for a chosen radius). 1 mark for area of $PQRS = 2r^2$. 1 mark for a complete method leading to $\frac{\pi-2}{4}$.

7 (4 marks)

The diagram shows an isosceles triangle ABC with $AC = BC = 16$ cm and $AB = 20$ cm. A semicircle has been cut from the triangle.

The centre of the semicircle is M , the midpoint of AB .

The diameter of the semicircle is 12 cm.

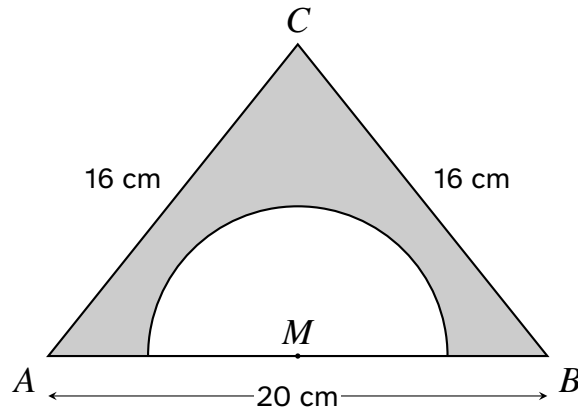


Diagram **NOT** accurately drawn

Work out the area of the shaded region.

Give your answer correct to 3 significant figures.

Shaded region = triangle – semicircle

The triangle is isosceles, so CM is its perpendicular height and $AM = 10$ cm.

Pythagoras in triangle AMC : $CM^2 = 16^2 - 10^2 = 156$

$CM = \sqrt{156} = 12.48 \dots$ cm

Area of triangle = $\frac{1}{2} \times 20 \times 12.48 \dots = 124.89 \dots \text{ cm}^2$

Semicircle: radius = $12 \div 2 = 6$ cm, area = $\frac{1}{2} \times \pi \times 6^2 = 56.54 \dots \text{ cm}^2$

Shaded = $124.89 \dots - 56.54 \dots = 68.35 \dots$

= 68.4 cm^2 (3 s.f.)

Answer: 68.4 cm^2

Tip: 16 cm is a sloping side, not the height. Use Pythagoras with half of AB to find the height first.

Marks: 1 mark for $16^2 - 10^2$. 1 mark for the triangle area ($124.9 \dots$). 1 mark for the semicircle area ($56.5 \dots$). 1 mark for 68.4.

8 (5 marks)

The diagram shows a flower bed.

It is made from a rectangle and a quarter circle with centre O .

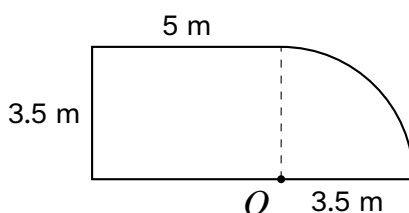


Diagram **NOT** accurately drawn

Ines wants to put edging around the whole edge of the flower bed.

Edging is sold in 2 m lengths.

Each length costs £3.75

Ines has £40

Does Ines have enough money to buy all the edging she needs?

You must show all your working.

The edge is made of three straight edges and one quarter circle arc.

$$\text{Arc} = \frac{1}{4} \times 2 \times \pi \times 3.5 = 5.497 \dots \text{ m}$$

The bottom edge is $5 + 3.5 = 8.5 \text{ m}$.

Straight edges = $5 + 3.5 + 8.5 = 17 \text{ m}$ (top, left side and bottom)

Total edge = $17 + 5.497 \dots = 22.49 \dots \text{ m}$

Lengths needed = $22.49 \dots \div 2 = 11.2 \dots$, so she needs 12 lengths.

Cost = $12 \times 3.75 = \text{£}45$

£45 is more than £40.

Answer: No: she needs 12 lengths, costing £45

Tip: Only the curved part is a quarter of a circumference. Include the bottom edge as $5 + 3.5 = 8.5 \text{ m}$, and round the number of lengths **up**.

Marks: 1 mark for the arc (5.49 ...). 1 mark for the total edge (22.4 ... or 22.5). 1 mark for 12 lengths. 1 mark for £45. 1 mark for “No” with correct figures.