

Arcs, Sectors and Segments

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

The diagram shows a sector OPQ of a circle, centre O .

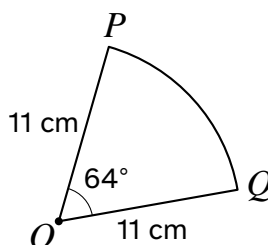


Diagram **NOT** accurately drawn

Work out the length of arc PQ .

Give your answer correct to 3 significant figures.

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{64}{360} \times 2 \times \pi \times 11$$

$$= 12.287 \dots$$

$$= 12.3 \text{ (3 s.f.)}$$

Answer: 12.3 cm

Tip: Arc length uses $2\pi r$ (the circumference), not πr^2 or πr .

Marks: 1 mark for $\frac{64}{360} \times 2 \times \pi \times 11$. 1 mark for 12.3.

2 (2 marks)

The arc length of a sector is 5π cm and the radius is 9 cm.
Find the angle of the sector.

Let the angle be θ .

$$\frac{\theta}{360} \times 2 \times \pi \times 9 = 5\pi$$

$$\frac{\theta}{360} \times 18\pi = 5\pi$$

Divide both sides by 18π : $\frac{\theta}{360} = \frac{5}{18}$

$$\theta = \frac{5}{18} \times 360 = 100$$

Answer: 100°

Marks: 1 mark for $\frac{\theta}{360} \times 18\pi = 5\pi$. 1 mark for 100.

3 (1 mark)

Show that the area of a sector of radius r and angle 45° is $\frac{\pi r^2}{8}$

$$\text{Sector area} = \frac{45}{360} \times \pi r^2$$

$$\frac{45}{360} = \frac{1}{8}$$

$$\text{So the area} = \frac{1}{8} \times \pi r^2 = \frac{\pi r^2}{8}$$

Answer: Shown

4 (3 marks)

The diagram shows a flower bed in the shape of a sector of a circle, centre O , radius 10 m.

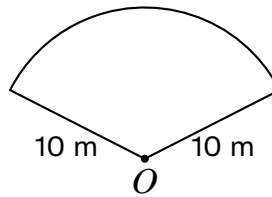


Diagram **NOT** accurately drawn

The area of the flower bed is $35\pi \text{ m}^2$

Work out, in terms of π , the length of the curved edge of the flower bed.

First find what fraction of a full circle the sector is.

$$\text{Area of the full circle} = \pi \times 10^2 = 100\pi$$

$$\text{Fraction} = \frac{35\pi}{100\pi} = \frac{7}{20}$$

$$\text{Circumference of the full circle} = 2 \times \pi \times 10 = 20\pi$$

$$\text{Curved edge} = \frac{7}{20} \times 20\pi = 7\pi$$

Answer: $7\pi \text{ m}$

Tip: The same fraction of the circle applies to the area and to the arc. You do not need the angle, but if you find it (126°) that works too.

Marks: 1 mark for the fraction $\frac{7}{20}$ (or the angle 126°). 1 mark for $\frac{7}{20} \times 20\pi$. 1 mark for 7π .

5 (8 marks)

The diagram shows a sector of a circle, centre O , radius 10 cm, with angle 120°
The shaded region is a segment.

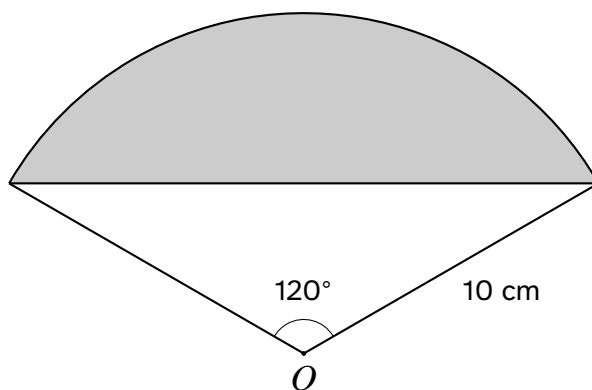


Diagram **NOT** accurately drawn

- (a) Calculate the area of the shaded segment.
Give your answer correct to 3 significant figures.

Segment = sector – triangle

$$\text{Sector} = \frac{120}{360} \times \pi \times 10^2 = 104.71 \dots$$

Use area of a triangle = $\frac{1}{2}ab \sin C$:

$$\text{Triangle} = \frac{1}{2} \times 10 \times 10 \times \sin 120^\circ = 43.30 \dots$$

$$\text{Segment} = 104.71 \dots - 43.30 \dots = 61.41 \dots$$

$$= 61.4 \text{ (3 s.f.)}$$

Answer: 61.4 cm²

Tip: Finding the sector and the triangle is not the end. Subtract them to get the segment.

Marks: 1 mark for the sector area. 1 mark for the triangle area. 1 mark for subtracting. 1 mark for 61.4.

- (b) Calculate the perimeter of the shaded segment.

The perimeter of the segment is the arc plus the chord.

$$\text{Arc} = \frac{120}{360} \times 2 \times \pi \times 10 = 20.94 \dots$$

Find the chord: split the triangle into two right-angled triangles.

Each has hypotenuse 10 and angle 60° at O .

Half the chord $= 10 \sin 60^\circ = 8.660 \dots$

Chord $= 2 \times 8.660 \dots = 17.32 \dots$

Perimeter $= 20.94 \dots + 17.32 \dots = 38.26 \dots$

$= 38.3$ (3 s.f.)

Answer: 38.3 cm

Tip: You can also find the chord with the cosine rule:

$$c^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos 120^\circ = 300.$$

Marks: 1 mark for the arc length. 1 mark for a correct method for the chord (e.g. the cosine rule or $10 \sin 60^\circ$). 1 mark for adding the arc and chord. 1 mark for 38.3.

6 (4 marks)

The diagram shows sector OPQ . O is the centre of the circle.
Angle $POQ = 140^\circ$

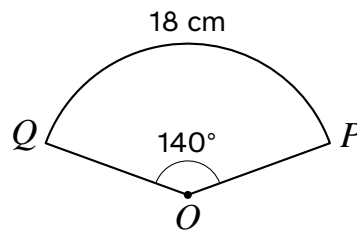


Diagram **NOT** accurately drawn

The length of arc PQ is 18 cm.

Work out the area of sector OPQ .

Give your answer correct to 3 significant figures.

First find the radius, r .

$$\frac{140}{360} \times 2 \times \pi \times r = 18$$

$$r = \frac{18 \times 360}{140 \times 2\pi} = 7.366 \dots \text{ cm}$$

$$\text{Area} = \frac{140}{360} \times \pi \times (7.366 \dots)^2$$

$$= 66.29 \dots$$

$$= 66.3 \text{ (3 s.f.)}$$

Answer: 66.3 cm^2

Tip: Keep the full calculator value of r . Rounding it to 7.4 first gives 66.9, which is not accurate enough.

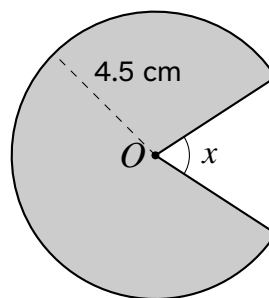
Marks: 1 mark for $\frac{140}{360} \times 2\pi r = 18$. 1 mark for $r = 7.36 \dots$ 1 mark for $\frac{140}{360} \times \pi \times r^2$. 1 mark for 66.3.

7 (3 marks)

A logo is made from a circle, centre O , radius 4.5 cm.

A sector with angle x is cut out of the circle.

The shaded shape is the logo.



*Diagram **NOT** accurately drawn*

The area of the logo is 52 cm^2

Work out the size of angle x .

Give your answer correct to the nearest degree.

$$\text{Area of the full circle} = \pi \times 4.5^2 = 63.61 \dots \text{ cm}^2$$

$$\text{Area of the missing sector} = 63.61 \dots - 52 = 11.61 \dots \text{ cm}^2$$

$$\frac{x}{360} \times 63.61 \dots = 11.61 \dots$$

$$x = \frac{11.61 \dots}{63.61 \dots} \times 360 = 65.74 \dots$$

= 66° (nearest degree)

Answer: 66°

Tip: The logo is the large sector, not the missing piece. Either find the missing area first, or solve $\frac{360-x}{360} \times \pi \times 4.5^2 = 52$.

Marks: 1 mark for $\pi \times 4.5^2$. 1 mark for a correct equation for x (e.g. $\frac{360-x}{360} \times \pi \times 4.5^2 = 52$). 1 mark for 66.

8 (4 marks)

OPQ is a sector of a circle, centre O , radius 9 cm.

Angle $POQ = 50^\circ$

R is the point on OQ such that PR is perpendicular to OQ .

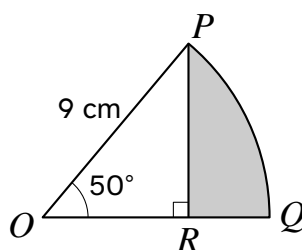


Diagram **NOT** accurately drawn

Calculate the area of the shaded region.

Give your answer correct to 3 significant figures.

Shaded region = sector OPQ – triangle OPR

$$\text{Sector} = \frac{50}{360} \times \pi \times 9^2 = 35.34 \dots$$

Triangle OPR is right-angled at R with hypotenuse $OP = 9$ cm.

$$OR = 9 \cos 50^\circ = 5.785 \dots$$

$$PR = 9 \sin 50^\circ = 6.894 \dots$$

$$\text{Triangle} = \frac{1}{2} \times 5.785 \dots \times 6.894 \dots = 19.94 \dots$$

$$\text{Shaded region} = 35.34 \dots - 19.94 \dots = 15.40 \dots$$

$$= 15.4 \text{ (3 s.f.)}$$

Answer: 15.4 cm^2

Tip: Triangle OPR is not the triangle OPQ . Use the right angle at R and SOHCAHTOA to find OR and PR first.

Marks: 1 mark for the sector area. 1 mark for OR or PR using trigonometry. 1 mark for the area of triangle OPR . 1 mark for 15.4.
