

# Angles in Parallel Lines and Polygons - Higher

## Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

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Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

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**1** (2 marks)

A regular polygon has 20 sides.

(a) Work out the size of each exterior angle.

The exterior angles of any polygon add up to  $360^\circ$ .

The polygon is regular, so they are all equal.

$$360 \div 20 = 18$$

**Answer:**  $18^\circ$

(b) Work out the size of each interior angle.

Interior angle + exterior angle =  $180^\circ$  (angles on a straight line).

$$180 - 18 = 162$$

**Answer:**  $162^\circ$

**Tip:** Find the exterior angle first. It is much quicker than using the angle sum.

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**2** (3 marks)

The ratio of the exterior angle to the interior angle of a regular polygon is 1:4  
Work out the number of sides of the polygon.

The exterior angle and the interior angle add up to  $180^\circ$ .

Share  $180^\circ$  in the ratio 1:4.

$1 + 4 = 5$  parts, so 1 part =  $180 \div 5 = 36^\circ$

Exterior angle =  $36^\circ$

Number of sides =  $360 \div 36 = 10$

**Answer:** 10

**Tip:** Share  $180^\circ$  (angles on a straight line), not  $360^\circ$ , in the ratio 1:4.

*Marks:* 1 mark for  $180 \div 5$  (or 36). 1 mark for  $360 \div 36$ . 1 mark for 10.

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**3** (3 marks)

The interior angles of a hexagon are  $x^\circ$ ,  $(x + 10)^\circ$ ,  $(x + 20)^\circ$ ,  $(x + 30)^\circ$ ,  $(x + 40)^\circ$  and  $(x + 50)^\circ$

Show that the smallest angle is  $95^\circ$

A hexagon has 6 sides.

Angle sum =  $(6 - 2) \times 180 = 720^\circ$

Add the angles:  $6x + 150$

So  $6x + 150 = 720$

$6x = 570$

$x = 95$

The smallest angle is  $x^\circ$ , so it is  $95^\circ$ .

**Answer:** Shown:  $x = 95$ , so the smallest angle is  $95^\circ$ .

**Tip:** Only a quadrilateral has angles adding up to  $360^\circ$ . For any polygon use  $(n - 2) \times 180^\circ$ .

*Marks:* 1 mark for 720. 1 mark for  $6x + 150 = 720$ . 1 mark for completing the working to reach  $x = 95$ .

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**4** (3 marks)

$ABC$  and  $DEF$  are parallel lines.  $BE$  is a straight line.

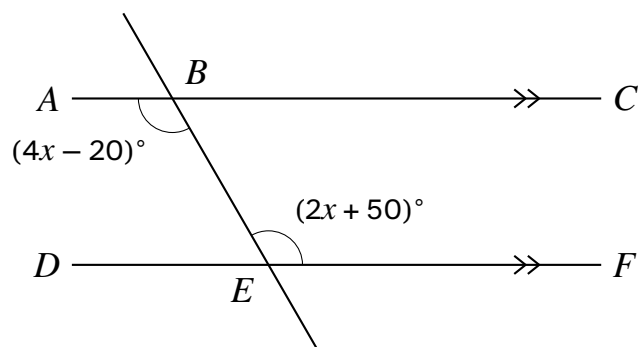


Diagram **NOT** accurately drawn

Work out the value of  $x$ .

Give a reason for your answer.

Angles  $ABE$  and  $BEF$  are alternate angles, and alternate angles are equal.

$$\text{So } 4x - 20 = 2x + 50$$

$$\text{Subtract } 2x \text{ from both sides: } 2x - 20 = 50$$

$$\text{Add } 20 \text{ to both sides: } 2x = 70$$

$$x = 35$$

**Answer:**  $x = 35$  (alternate angles are equal)

**Tip:** Write the reason in full: “alternate angles are equal”. “Z-angles” does not get the mark.

**Marks:** 1 mark for  $4x - 20 = 2x + 50$ . 1 mark for 35. 1 mark for the reason “alternate angles are equal”.

5 (3 marks)

$ABCDE$  is a regular pentagon.

$ABF$  is an equilateral triangle, with  $F$  inside the pentagon.

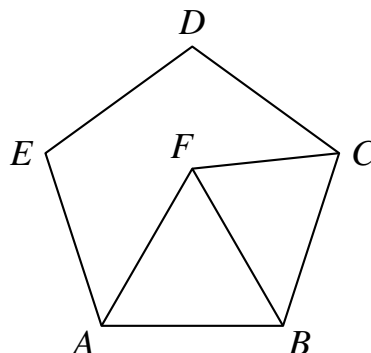


Diagram **NOT** accurately drawn

Work out the size of angle  $BFC$ .

You must show all your working.

Interior angle of a regular pentagon  $= 180 - 360 \div 5 = 180 - 72 = 108^\circ$ , so angle  $ABC = 108^\circ$

Triangle  $ABF$  is equilateral, so angle  $ABF = 60^\circ$  and  $BF = AB$ .

Angle  $FBC = 108 - 60 = 48^\circ$

$BF = AB = BC$ , so triangle  $BFC$  is isosceles, with angle  $BFC = \text{angle } BCF$ .

Angle  $BFC = (180 - 48) \div 2 = 66^\circ$

**Answer:**  $66^\circ$

**Tip:** Look for equal sides:  $BF$  is a side of the equilateral triangle and  $BC$  is a side of the pentagon, so they are the same length.

Marks: 1 mark for 108. 1 mark for 48. 1 mark for 66.

6 (3 marks)

$AB$ ,  $BC$  and  $CD$  are three sides of a regular polygon.

The sides  $AB$  and  $DC$  are extended to meet at  $X$ .

Angle  $BXC = 140^\circ$

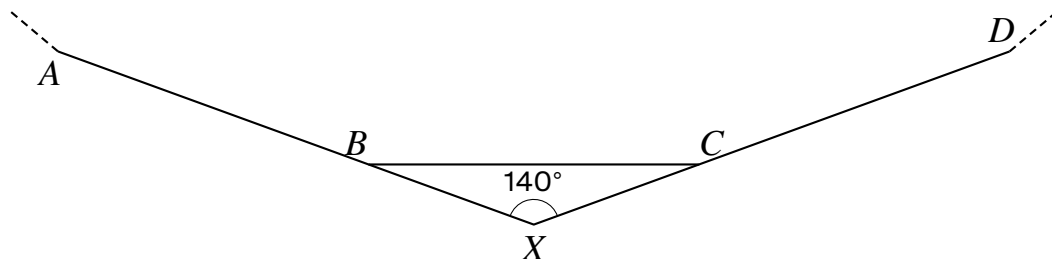


Diagram **NOT** accurately drawn

Work out the number of sides of the polygon.

You must show all your working.

Angles  $XBC$  and  $XCB$  are on straight lines with interior angles of the polygon, so they are both exterior angles.

The polygon is regular, so they are equal.

Angles in a triangle add up to  $180^\circ$ .

Each exterior angle =  $(180 - 140) \div 2 = 20^\circ$

Number of sides =  $360 \div 20 = 18$

**Answer:** 18

**Tip:** Angle  $XBC$  is on a straight line with the interior angle  $ABC$ , so it is an **exterior** angle of the polygon.

*Marks:* 1 mark for  $(180 - 140) \div 2$ . 1 mark for  $360 \div 20$ . 1 mark for 18.

**7** (4 marks)

$ABC$  is an isosceles triangle with  $AB = AC$ .

$PAQ$  is a straight line, parallel to  $BC$ .

Angle  $PAB = 52^\circ$

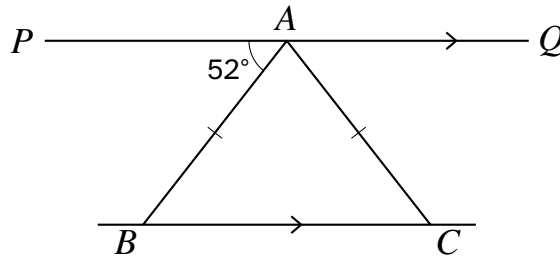


Diagram **NOT** accurately drawn

Work out the size of angle  $BAC$ .

Give a reason for each stage of your working.

Angle  $ABC = 52^\circ$

Reason: alternate angles are equal ( $PQ$  is parallel to  $BC$ ).

Angle  $ACB = 52^\circ$

Reason: base angles of an isosceles triangle are equal ( $AB = AC$ ).

Angle  $BAC = 180 - 52 - 52 = 76^\circ$

Reason: angles in a triangle add up to  $180^\circ$ .

**Answer:**  $76^\circ$

**Tip:** Give a reason for every angle you find, not just the last one. Write each angle on the diagram as you go.

*Marks:* 1 mark for angle  $ABC = 52$ . 1 mark for angle  $ACB = 52$ . 1 mark for 76. 1 mark for giving all the reasons correctly.

8 (4 marks)

$A, B, C$  and  $D$  are four vertices next to each other on a regular polygon with  $n$  sides.

$AC$  is a straight line.

Angle  $BAC = 12^\circ$

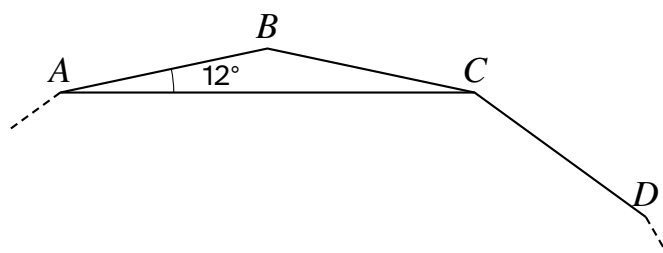


Diagram **NOT** accurately drawn

Work out the value of  $n$ .

You must show all your working.

$AB = BC$  (sides of a regular polygon), so triangle  $ABC$  is isosceles.

So angle  $BCA = \text{angle } BAC = 12^\circ$

Interior angle  $ABC = 180 - 12 - 12 = 156^\circ$

Exterior angle  $= 180 - 156 = 24^\circ$

Number of sides  $n = 360 \div 24 = 15$

**Answer:** 15

**Tip:** Triangle  $ABC$  is isosceles because two of its sides are sides of the regular polygon. Use this to find the interior angle first.

*Marks:* 1 mark for angle  $BCA = 12^\circ$ . 1 mark for 156. 1 mark for  $360 \div 24$ . 1 mark for 15.