

3D Pythagoras and Trigonometry

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

ABCDEFGH is a cuboid.

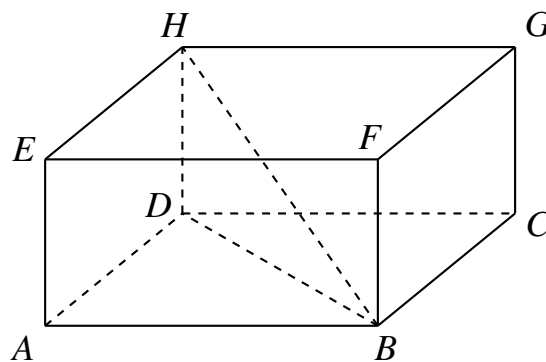


Diagram **NOT** accurately drawn

$$DH = 6 \text{ cm}$$

The angle between BH and the plane $ABCD$ is 35°

Work out the length of BH .

Give your answer correct to 3 significant figures.

H is vertically above D , so BD is the “shadow” of BH on the base.

So angle $HBD = 35^\circ$, and triangle BDH has a right angle at D .

DH is opposite the 35° angle and BH is the hypotenuse.

$$\sin 35^\circ = \frac{6}{BH}$$

$$BH = \frac{6}{\sin 35^\circ} = 10.46 \dots$$

Answer: 10.5 cm

Tip: BH is the hypotenuse, so divide by $\sin 35^\circ$. $6 \sin 35^\circ = 3.44$ is a common wrong answer.

Marks: 1 mark for $\frac{6}{\sin 35^\circ}$. 1 mark for 10.5.

2 (3 marks)

Will a 45 cm stick fit inside a box measuring 30 cm by 25 cm by 20 cm?
You must show your working.

The longest stick that fits goes corner to opposite corner (the space diagonal).

$$d^2 = 30^2 + 25^2 + 20^2 = 900 + 625 + 400 = 1925$$

$$d = \sqrt{1925} = 43.87 \dots \text{ cm}$$

$$43.87 \dots < 45$$

So the stick will not fit.

Answer: No. The longest diagonal is 43.9 cm, which is less than 45 cm.

Marks: 1 mark for $30^2 + 25^2 + 20^2$. 1 mark for 43.9. 1 mark for “No” with a comparison of 43.9 and 45.

3 (4 marks)

$ABCDEFGH$ is a cuboid.

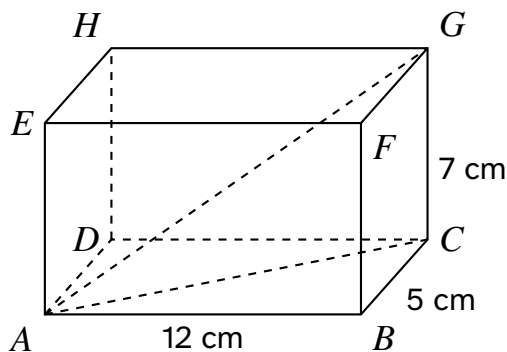


Diagram **NOT** accurately drawn

Calculate the size of the angle between AG and the plane $ABCD$.
Give your answer correct to 1 decimal place.

Step 1: find the base diagonal AC (right angle at B).

$$AC = \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ cm}$$

Step 2: G is vertically above C , so the angle we want is angle GAC . Call it θ .

Triangle ACG has a right angle at C .

Opposite = $CG = 7$, adjacent = $AC = 13$

$$\tan \theta = \frac{7}{13}$$

$$\theta = \tan^{-1}\left(\frac{7}{13}\right) = 28.30 \dots$$

Answer: 28.3°

Tip: The angle with the plane $ABCD$ is between AG and its “shadow” AC on the base, not between AG and the edge AB .

Marks: 1 mark for finding AC . 1 mark for identifying angle GAC . 1 mark for $\tan \theta = \frac{7}{13}$. 1 mark for 28.3.

4 (4 marks)

A pole stands vertically at a corner of a rectangular field $ABCD$.

$AB = 40$ m and $BC = 30$ m. The pole is at A and is 15 m tall. T is the top of the pole.

Calculate the size of angle TCA .

Give your answer correct to 1 decimal place.

Sketch the field with the pole at A .

Step 1: C is the opposite corner, so find the diagonal AC .

$$AC = \sqrt{40^2 + 30^2} = \sqrt{2500} = 50 \text{ m}$$

Step 2: the pole, AC and CT make a right-angled triangle, with the right angle at A .

Opposite = 15 (the pole), adjacent = 50

$$\tan TCA = \frac{15}{50} = 0.3$$

$$\text{Angle } TCA = \tan^{-1}(0.3) = 16.69 \dots$$

Answer: 16.7°

Marks: 1 mark for $40^2 + 30^2$. 1 mark for $AC = 50$. 1 mark for $\tan TCA = \frac{15}{50}$. 1 mark for 16.7.

5 (5 marks)

The diagram shows a cuboid $ABCDEFGH$.

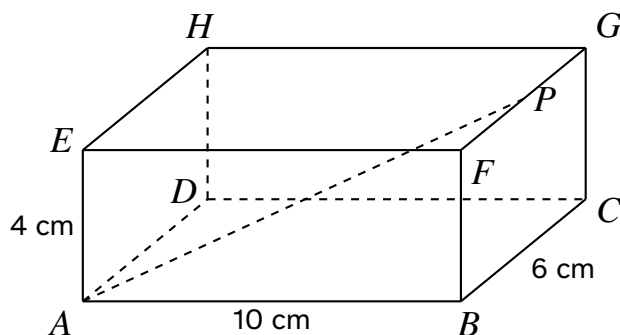


Diagram **NOT** accurately drawn

$AB = 10 \text{ cm}$ $BC = 6 \text{ cm}$ $AE = 4 \text{ cm}$

P is the midpoint of the edge FG .

Calculate the size of the angle between the line AP and the plane $ABCD$.

Give your answer correct to 1 decimal place.

Let Q be the midpoint of BC . Q is directly below P , so $PQ = 4 \text{ cm}$.

AQ is the “shadow” of AP on the base, so the angle we want is angle PAQ .

$$BQ = 6 \div 2 = 3 \text{ cm}$$

Triangle ABQ has a right angle at B : $AQ = \sqrt{10^2 + 3^2} = \sqrt{109} = 10.44 \dots \text{ cm}$

Triangle AQP has a right angle at Q : opposite = $PQ = 4$, adjacent = AQ

$$\tan PAQ = \frac{4}{10.44 \dots}$$

$$\text{Angle } PAQ = 20.96 \dots$$

Answer: 21.0°

Tip: Find the point on the base directly below P first. The angle with the base is always in a right-angled triangle with a vertical side.

Marks: 1 mark for $BQ = 3$ (or the point below P). 1 mark for $10^2 + 3^2$. 1 mark for $AQ = \sqrt{109}$. 1 mark for $\tan^{-1}\left(\frac{4}{AQ}\right)$. 1 mark for 21.0 .

6 (5 marks)

$VABCD$ is a solid pyramid with V vertically above the centre of the base.

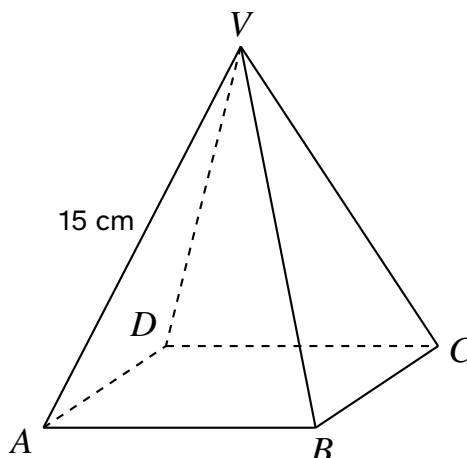


Diagram **NOT** accurately drawn

$ABCD$ is a square.

Each sloping edge of the pyramid has length 15 cm.

The angle between each sloping edge and the base $ABCD$ is 62°

Calculate the volume of the pyramid.

Give your answer correct to 3 significant figures.

(Volume of a pyramid = $\frac{1}{3} \times$ area of base $\times$ vertical height)

Let X be the centre of the base. Triangle VXA has a right angle at X and angle $VAX = 62^\circ$.

Height $VX = 15 \sin 62^\circ = 13.24 \dots$ cm

$AX = 15 \cos 62^\circ = 7.042 \dots$ cm

AX is half of the diagonal AC , so $AC = 14.08 \dots$ cm

Side of the square = $\frac{AC}{\sqrt{2}} = 9.959 \dots$ cm, so base area = $9.959 \dots^2 = 99.18 \dots \text{ cm}^2$

$$\text{Volume} = \frac{1}{3} \times 99.18 \dots \times 13.24 \dots = 437.8 \dots$$

Answer: 438 cm³

Tip: AX is half of the diagonal of the square, not half of a side. Keep full calculator values until the end.

Marks: 1 mark for $15 \sin 62^\circ$. 1 mark for $15 \cos 62^\circ$. 1 mark for the side or area of the base. 1 mark for $\frac{1}{3} \times \text{base} \times \text{height}$. 1 mark for 438.
