

Vectors

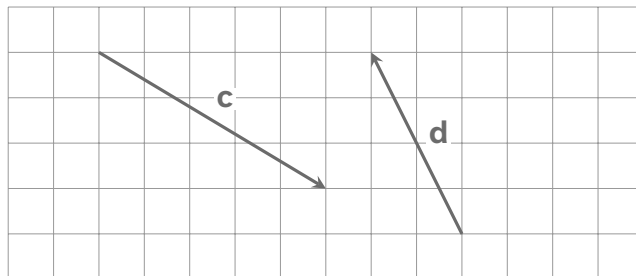
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

The vectors **c** and **d** are shown on the grid.



(a) Write **c** as a column vector.

Count squares from the start of the arrow to the end.

5 squares right, so the top number is 5.

3 squares down, so the bottom number is -3 .

Answer: $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

(b) Write **d** as a column vector.

2 squares left, so the top number is -2 .

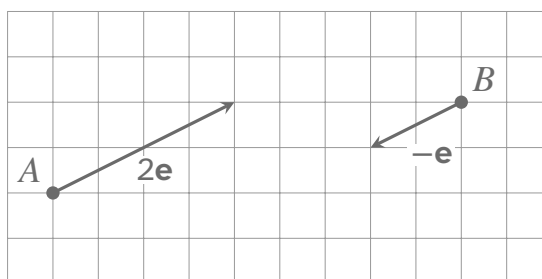
4 squares up, so the bottom number is 4.

Answer: $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

Tip: Top number is left/right, bottom number is up/down. Follow the direction of the arrow: left and down are negative.

2 (2 marks)

$$\mathbf{e} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$



- (a) On the grid, draw and label the vector $2\mathbf{e}$ starting at point A .

$$2\mathbf{e} = 2 \times \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

Draw an arrow from A going 4 right and 2 up, and label it $2\mathbf{e}$.

Answer: Arrow $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Tip: Start the arrow exactly at A , put an arrowhead on it, and double **both** movements.

- (b) On the grid, draw and label the vector $-\mathbf{e}$ starting at point B .

$-\mathbf{e}$ is the same length as $\mathbf{e}$ but in the opposite direction.

$$-\mathbf{e} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

Draw an arrow from B going 2 left and 1 down.

Answer: Arrow $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

3 (4 marks)

$$\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

(a) Work out $\mathbf{a} + \mathbf{b}$ as a column vector.

Add the top numbers: $3 + (-1) = 2$

Add the bottom numbers: $-2 + 6 = 4$

Answer: $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

Tip: A column vector is not a fraction. Keep the top and bottom numbers separate, and never draw a fraction line.

(b) Work out $4\mathbf{a}$ as a column vector.

Multiply both numbers by 4.

$$4 \times 3 = 12$$

$$4 \times (-2) = -8$$

Answer: $\begin{pmatrix} 12 \\ -8 \end{pmatrix}$

(c) Work out $2\mathbf{a} - \mathbf{b}$ as a column vector.

$$2\mathbf{a} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

$$2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

$$\text{Top: } 6 - (-1) = 6 + 1 = 7$$

Bottom: $-4 - 6 = -10$

Answer: $\begin{pmatrix} 7 \\ -10 \end{pmatrix}$

Tip: Double **both** numbers in **a** (doubling, not squaring). Then subtract carefully:

$$6 - (-1) = 7.$$

Marks: 1 mark for $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$. 1 mark for the correct answer.

4 (2 marks)

$$\mathbf{f} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \quad \mathbf{g} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Work out $3\mathbf{f} + 2\mathbf{g}$ as a column vector.

$$3\mathbf{f} = \begin{pmatrix} -3 \\ 12 \end{pmatrix}$$

$$2\mathbf{g} = \begin{pmatrix} 10 \\ -4 \end{pmatrix}$$

$$\text{Top: } -3 + 10 = 7$$

$$\text{Bottom: } 12 + (-4) = 8$$

Answer: $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$

Marks: 1 mark for $\begin{pmatrix} -3 \\ 12 \end{pmatrix}$ or $\begin{pmatrix} 10 \\ -4 \end{pmatrix}$. 1 mark for the correct answer.

5 (1 mark)

Which of these vectors is parallel to $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$?

Tick the vector.

Parallel vectors are multiples of each other.

$$3 \times 2 = 6 \text{ and } 3 \times (-3) = -9$$

So $\begin{pmatrix} 6 \\ -9 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, which is parallel.

Answer: $\begin{pmatrix} 6 \\ -9 \end{pmatrix}$

6 (2 marks)

$$\overrightarrow{PQ} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

P is the point $(2, 5)$.

Work out the coordinates of Q .

Start at P and move by the vector $\overrightarrow{PQ}$: 4 right and 3 down.

$$x: 2 + 4 = 6$$

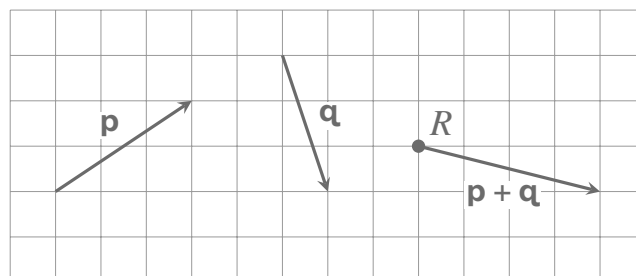
$$y: 5 + (-3) = 2$$

Answer: $(6, 2)$

Marks: 1 mark for a correct method, e.g. $(2 + 4, 5 - 3)$. 1 mark for the correct answer.

7 (2 marks)

The vectors $\mathbf{p}$ and $\mathbf{q}$ are drawn on the grid.



From the point R , draw the vector $\mathbf{p} + \mathbf{q}$

Read each vector from the grid.

$\mathbf{p}$: 3 right and 2 up, so $\mathbf{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\mathbf{q}$: 1 right and 3 down, so $\mathbf{q} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$$\mathbf{p} + \mathbf{q} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

Draw an arrow from R going 4 right and 1 down.

Answer: Arrow $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ from R

Tip: You can also draw $\mathbf{p}$ from R , then $\mathbf{q}$ from the end of $\mathbf{p}$. The vector $\mathbf{p} + \mathbf{q}$ goes from the start of $\mathbf{p}$ straight to the end of $\mathbf{q}$. Remember the arrowhead.

Marks: 1 mark for $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ or an arrow from R with one correct movement. 1 mark for the correct arrow from R .

8 (3 marks)

$ABCD$ is a trapezium.

DC is parallel to AB , and DC is 3 times as long as AB .

$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$

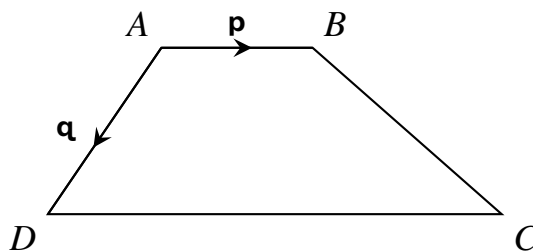


Diagram **NOT** accurately drawn

(a) Find, in terms of $\mathbf{p}$, the vector $\overrightarrow{DC}$

DC is parallel to AB and goes in the same direction (left to right).

DC is 3 times as long as AB .

$$\overrightarrow{DC} = 3\mathbf{p}$$

Answer: $3\mathbf{p}$

- (b) Find, in terms of $\mathbf{p}$ and $\mathbf{q}$, the vector $\overrightarrow{BD}$

Go from B to A , then from A to D .

$$\overrightarrow{BA} = -\mathbf{p} \text{ (against the arrow) and } \overrightarrow{AD} = \mathbf{q}$$

$$\overrightarrow{BD} = -\mathbf{p} + \mathbf{q}$$

Answer: $\mathbf{q} - \mathbf{p}$

Tip: Going against an arrow changes the sign. Write the route (for example $B \rightarrow A \rightarrow D$) before you write the vectors.

- (c) Find, in terms of $\mathbf{p}$ and $\mathbf{q}$, the vector $\overrightarrow{BC}$

Go from B to D , then from D to C .

$$\overrightarrow{BD} = -\mathbf{p} + \mathbf{q} \text{ and } \overrightarrow{DC} = 3\mathbf{p}$$

$$\overrightarrow{BC} = -\mathbf{p} + \mathbf{q} + 3\mathbf{p}$$

$$= 2\mathbf{p} + \mathbf{q}$$

Answer: $2\mathbf{p} + \mathbf{q}$

9 (3 marks)

A is the point $(-1, 4)$.

$$\overrightarrow{AB} = 3\mathbf{u} - \mathbf{w} \text{ where } \mathbf{u} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \text{ and } \mathbf{w} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Work out the coordinates of B .

$$3\mathbf{u} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$$3\mathbf{u} - \mathbf{w} = \begin{pmatrix} 6 \\ -3 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\text{Top: } 6 - 3 = 3$$

Bottom: $-3 - 5 = -8$

So $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$

Start at $A(-1, 4)$ and move 3 right and 8 down:

$$B = (-1 + 3, 4 - 8) = (2, -4)$$

Answer: $(2, -4)$

Marks: 1 mark for $3\mathbf{u} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$. 1 mark for $\begin{pmatrix} 3 \\ -8 \end{pmatrix}$. 1 mark for $(2, -4)$.
