

Spheres, Cones and Pyramids

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

A sphere has radius 4.5 cm.

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

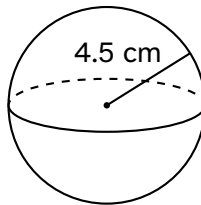


Diagram **NOT** accurately drawn

Work out the volume of the sphere.

Give your answer correct to 1 decimal place.

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

Here $r = 4.5$.

$$V = \frac{4}{3} \times \pi \times 4.5^3$$

$$= 381.70 \dots$$

Round to 1 decimal place: 381.7

Answer: 381.7 cm³

Marks: 1 mark for $\frac{4}{3} \times \pi \times 4.5^3$. 1 mark for 381.7.

2 (2 marks)

A solid hemisphere has diameter 10 cm.
Work out the volume of the hemisphere.
Give your answer correct to 3 significant figures.

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

The diameter is 10 cm, so the radius is $r = 5$ cm.

A hemisphere is half of a sphere.

$$\text{Volume} = \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$$

$$= 261.79 \dots$$

Round to 3 significant figures: 262

Answer: 262 cm³

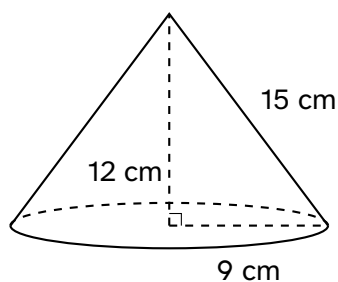
Tip: Use the radius (5 cm), not the diameter, and remember to halve for a hemisphere.

Marks: 1 mark for $\frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$. 1 mark for 262 (anything from 261.7 to 262).

3 (4 marks)

A cone has radius 9 cm and perpendicular height 12 cm.

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h \quad \text{Curved surface area of a cone} = \pi r l$$



*Diagram **NOT** accurately drawn*

- (a) Work out the volume of the cone.
Give your answer correct to 1 decimal place.

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

Use the **perpendicular** height, $h = 12$.

$$V = \frac{1}{3} \times \pi \times 9^2 \times 12$$

$$= 324\pi = 1017.87 \dots$$

Round to 1 decimal place: 1017.9

Answer: 1017.9 cm³

Marks: 1 mark for $\frac{1}{3} \times \pi \times 9^2 \times 12$. 1 mark for 1017.9.

- (b) The slant height of the cone is 15 cm.

Work out the curved surface area of the cone.

Give your answer correct to 1 decimal place.

Curved surface area = $\pi r l$, where l is the **slant** height.

$$= \pi \times 9 \times 15$$

$$= 135\pi = 424.11 \dots$$

Round to 1 decimal place: 424.1

Answer: 424.1 cm²

Tip: Volume uses the perpendicular height (12 cm). Curved surface area uses the slant height (15 cm). Do not mix them up.

Marks: 1 mark for $\pi \times 9 \times 15$. 1 mark for 424.1.

4 (3 marks)

A pyramid has a rectangular base 8 cm by 9 cm.

The volume of the pyramid is 240 cm³

Work out the perpendicular height of the pyramid.

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$\text{Area of base} = 8 \times 9 = 72 \text{ cm}^2$$

$$\text{Volume} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$\text{So } \frac{1}{3} \times 72 \times h = 240$$

$$\frac{1}{3} \times 72 = 24, \text{ so } 24h = 240$$

$$h = 240 \div 24 = 10$$

Answer: 10 cm

Tip: Keep the $\frac{1}{3}$ in the formula. Without it you would get $240 \div 72 = 3.33 \dots$, which is wrong.

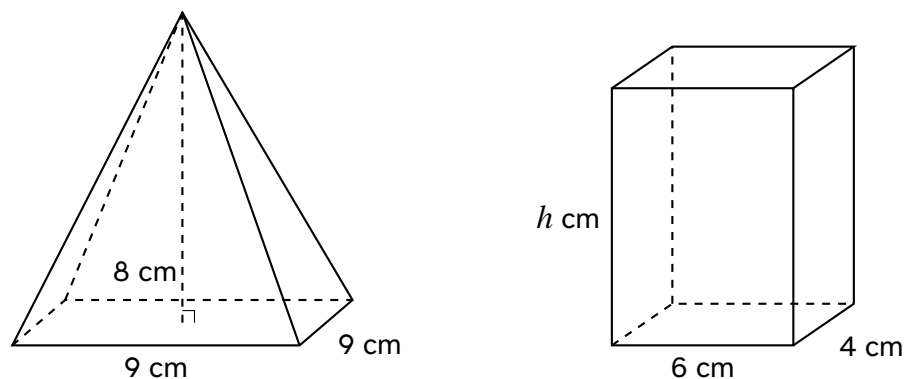
Marks: 1 mark for 72. 1 mark for $24h = 240$. 1 mark for 10.

5 (3 marks)

The diagram shows a solid square-based pyramid and a solid cuboid.

The pyramid has base side 9 cm and perpendicular height 8 cm.

The cuboid has a base 6 cm by 4 cm and height h cm.



*Diagram **NOT** accurately drawn*

The pyramid and the cuboid have the same volume.

Work out the height, h cm, of the cuboid.

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$\text{Area of the pyramid's base} = 9 \times 9 = 81 \text{ cm}^2$$

$$\text{Volume of the pyramid} = \frac{1}{3} \times 81 \times 8 = 216 \text{ cm}^3$$

$$\text{Volume of the cuboid} = 6 \times 4 \times h = 24h$$

$$\text{The volumes are equal, so } 24h = 216$$

$$h = 216 \div 24 = 9$$

Answer: $h = 9$

Tip: Do not forget the $\frac{1}{3}$ for the pyramid. $9 \times 9 \times 8 = 648$ would be the volume of a cuboid.

Marks: 1 mark for $\frac{1}{3} \times 81 \times 8 (= 216)$. 1 mark for $6 \times 4 \times h = 216$. 1 mark for 9.

6 (3 marks)

A solid hemisphere has radius 7 cm.

Work out the total surface area of the hemisphere.

Give your answer correct to 1 decimal place.

$$\text{Surface area of a sphere} = 4\pi r^2$$

A solid hemisphere has a curved surface and a flat circle.

$$\text{Curved surface} = \frac{1}{2} \times 4 \times \pi \times 7^2 = 98\pi$$

$$\text{Flat circle} = \pi \times 7^2 = 49\pi$$

$$\text{Total} = 98\pi + 49\pi = 147\pi$$

$$147\pi = 461.81\dots$$

Round to 1 decimal place: 461.8

Answer: 461.8 cm²

Tip: Remember the flat circular face. Half of $4\pi r^2$ is only the curved part.

Marks: 1 mark for 98π . 1 mark for adding 49π . 1 mark for 461.8.

7 (4 marks)

A solid metal sphere has radius 3 cm.

The sphere is melted down and made into a solid cylinder of radius 2 cm.

No metal is wasted.

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

Work out the height of the cylinder.

No metal is wasted, so the cylinder has the same volume as the sphere.

$$\text{Volume of sphere} = \frac{4}{3} \times \pi \times 3^3 = 36\pi$$

$$\text{Volume of cylinder} = \pi r^2 h = \pi \times 2^2 \times h = 4\pi h$$

$$\text{So } 4\pi h = 36\pi$$

$$\text{Divide both sides by } 4\pi: h = 9$$

Answer: 9 cm

Marks: 1 mark for 36π . 1 mark for $4\pi h$. 1 mark for putting them equal. 1 mark for 9.

8 (4 marks)

The diagram shows a solid model rocket made from a cylinder and a cone.

The cylinder and the cone both have radius 2 cm.

The height of the cylinder is 9 cm. The height of the cone is 6 cm.

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

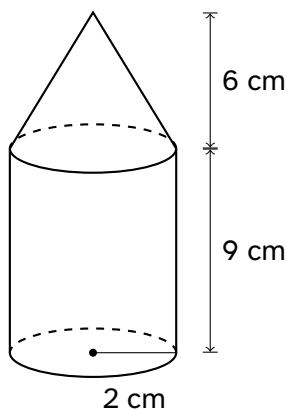


Diagram **NOT** accurately drawn

Work out the total volume of the model rocket.

Give your answer in terms of π .

You must show all your working.

Find the volume of each part in terms of π , then add.

Cylinder: $\pi \times 2^2 \times 9 = 36\pi$

Cone: $\frac{1}{3} \times \pi \times 2^2 \times 6 = 8\pi$

Total = $36\pi + 8\pi = 44\pi$

Answer: $44\pi \text{ cm}^3$

Tip: Use the height of **each part** with its own formula: 9 cm for the cylinder and 6 cm for the cone. Keep π as a symbol, do not multiply it out.

Marks: 1 mark for $\pi \times 2^2 \times 9$. 1 mark for $\frac{1}{3} \times \pi \times 2^2 \times 6$. 1 mark for adding the two volumes. 1 mark for 44π .