

Special Sequences

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Here is a list of numbers.

1 6 8 10 16 27 36 45 49

(a) Write down all the numbers from the list that are square numbers.

Square numbers: 1, 4, 9, 16, 25, 36, 49, ...

From the list: $1 = 1^2$, $16 = 4^2$, $36 = 6^2$, $49 = 7^2$

Answer: 1, 16, 36, 49

(b) Write down all the numbers from the list that are triangular numbers.

Triangular numbers: add 2, then 3, then 4, and so on.

1, 3, 6, 10, 15, 21, 28, 36, 45, ...

From the list: 1, 6, 10, 36, 45

Answer: 1, 6, 10, 36, 45

(c) Write down all the numbers from the list that are cube numbers.

Cube numbers: $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, ...

From the list: 1, 8, 27

Answer: 1, 8, 27

Tip: 1 is a square number, a triangular number **and** a cube number, so it goes in every answer.

2 (2 marks)

Here are the first five terms of a Fibonacci sequence.

2 9 16 25

- (a) Write down the missing term.

In a Fibonacci sequence each term is the sum of the two terms before it.

So $2 + \text{missing term} = 9$

Missing term: $9 - 2 = 7$

Check: $7 + 9 = 16$ and $9 + 16 = 25$ ✓

Answer: 7

- (b) Work out the 7th term of the sequence.

6th term: $16 + 25 = 41$

7th term: $25 + 41 = 66$

Answer: 66

Tip: The sequence shows 5 terms, so you need two more steps to reach the 7th term.

3 (2 marks)

Here are the first four terms of a geometric sequence.

2 6 18 54

Find the 6th term of this sequence.

$6 \div 2 = 3$, so you multiply by 3 each time.

5th term: $54 \times 3 = 162$

6th term: $162 \times 3 = 486$

Answer: 486

Marks: 1 mark for multiplying by 3. 1 mark for 486.

4 (1 mark)

Here are the first four terms of a geometric sequence.

32 16 8 4

Find the next term of this sequence.

Each term is half of the one before (multiply by $\frac{1}{2}$).

$$4 \div 2 = 2$$

Answer: 2

5 (2 marks)

The 9th triangle number is 45

Work out the 12th triangle number.

To get the next triangle number you add the next counting number.

The 9th triangle number is 45, so to get the 10th you add 10.

$$10\text{th: } 45 + 10 = 55$$

$$11\text{th: } 55 + 11 = 66$$

$$12\text{th: } 66 + 12 = 78$$

(Another way: the 12th triangle number is $1 + 2 + 3 + \dots + 12 = 78$.)

Answer: 78

Tip: To go from the 9th triangle number to the 10th you add 10, not 9. Label each term with its position as you go.

Marks: 1 mark for adding 10, 11 and 12 (or $1 + 2 + \dots + 12$). 1 mark for 78.

6 (2 marks)

Here are the first five terms of a sequence.

2 5 10 17 26

Find the next term of this sequence.

Find the differences between the terms:

3, 5, 7, 9

The differences go up by 2 each time, so the next difference is 11.

Next term: $26 + 11 = 37$

(These are the square numbers plus 1: $6^2 + 1 = 37$.)

Answer: 37

Marks: 1 mark for the differences 3, 5, 7, 9. 1 mark for 37.

7 (2 marks)

Here are the first three terms of a sequence.

5 10 20

- (a) Write down a possible 4th term of the sequence.
Give the rule you have used.

One rule: each term is double the term before.

$$5 \times 2 = 10, 10 \times 2 = 20$$

$$4\text{th term: } 20 \times 2 = 40$$

Answer: e.g. 40 (multiply by 2 each time)

- (b) Write down a **different** possible 4th term of the sequence.
Give the rule you have used.

Another rule: the differences are +5 and +10, and they go up by 5 each time.

$$\text{The next difference is +15, so the 4th term is } 20 + 15 = 35.$$

Or: add the two previous terms (Fibonacci). $10 + 20 = 30$

Answer: e.g. 35 (differences go up by 5) or 30 (add the two previous terms)

Tip: The first few terms of a sequence don't always tell you the rule. More than one rule can fit, so always say which rule you used.

8 (1 mark)

Joe says,

'5 10 20 35 is a geometric sequence.'

Is Joe correct?

You must give a reason for your answer.

In a geometric sequence you multiply by the same number each time.

$$10 \div 5 = 2 \text{ and } 20 \div 10 = 2$$

But $35 \div 20 = 1.75$, not 2.

So there is no common multiplier, and Joe is wrong.

Answer: No: $10 \div 5 = 2$ but $35 \div 20 = 1.75$, so there is no common ratio.

9 (2 marks)

The n th term of a sequence is $50 - 3n^2$

(a) Work out the 3rd term of this sequence.

Substitute $n = 3$ into $50 - 3n^2$.

Square first: $3^2 = 9$

Then multiply by 3: $3 \times 9 = 27$

$$50 - 27 = 23$$

Answer: 23

Tip: $3n^2$ means $3 \times n^2$. Square n first, then multiply by 3. Only the n is squared.

(b) Is 20 a term of this sequence?

Show how you get your answer.

Work out the terms: $n = 1$: 47, $n = 2$: 38, $n = 3$: 23, $n = 4$: 2

The terms go down, and 20 is between the 3rd term (23) and the 4th term (2).

So 20 is not a term.

(Or: $50 - 3n^2 = 20$ gives $n^2 = 10$, and 10 is not a square number.)

Answer: No. The 3rd term is 23 and the 4th term is 2, so 20 is not a term.

10 (3 marks)

A Fibonacci sequence starts 3 k ...

Its 6th term is 49

Find the value of k .

Add the two previous terms each time.

3rd term: $3 + k$

4th term: $k + (3 + k) = 3 + 2k$

5th term: $(3 + k) + (3 + 2k) = 6 + 3k$

6th term: $(3 + 2k) + (6 + 3k) = 9 + 5k$

The 6th term is 49, so $9 + 5k = 49$.

Subtract 9: $5k = 40$

Divide by 5: $k = 8$

Check: 3, 8, 11, 19, 30, 49 ✓

Answer: $k = 8$

Tip: Collect the numbers and the k terms separately at each step, e.g.

$(3 + 2k) + (6 + 3k) = 9 + 5k$.

Marks: 1 mark for continuing the sequence in terms of k . 1 mark for $9 + 5k = 49$. 1 mark for 8.

11 (5 marks)

The first two terms of a Fibonacci sequence are p and q

- (a) Show that the sum of the first five terms of this sequence is $5p + 7q$

Add the two previous terms each time.

3rd term: $p + q$

4th term: $q + (p + q) = p + 2q$

5th term: $(p + q) + (p + 2q) = 2p + 3q$

Sum: $p + q + (p + q) + (p + 2q) + (2p + 3q)$

p terms: $p + p + p + 2p = 5p$ q terms: $q + q + 2q + 3q = 7q$

So the sum is $5p + 7q$.

Marks: 1 mark for the 4th and 5th terms, $p + 2q$ and $2p + 3q$. 1 mark for adding all five terms to get $5p + 7q$.

- (b) The 4th term is 17 and the sum of the first five terms is 67
Work out the value of p and the value of q .

4th term: $p + 2q = 17$

Sum of first five terms: $5p + 7q = 67$

Multiply the first equation by 5: $5p + 10q = 85$

Subtract the second equation: $3q = 85 - 67 = 18$

So $q = 6$

$p + 2 \times 6 = 17$, so $p = 17 - 12 = 5$

Check: 5, 6, 11, 17, 28 and $5 + 6 + 11 + 17 + 28 = 67$ ✓

Answer: $p = 5$, $q = 6$

Tip: Use the expressions from part (a) to write two equations, then solve them simultaneously.

Marks: 1 mark for $p + 2q = 17$ and $5p + 7q = 67$. 1 mark for a correct method to find one letter. 1 mark for both values correct.