

Solving Quadratics by Factorising

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

- (a) Write down the two solutions of $(x - 6)(x + 1) = 0$

Two things multiply to give 0, so one of them must be 0.

$$x - 6 = 0 \text{ gives } x = 6$$

$$x + 1 = 0 \text{ gives } x = -1$$

Answer: $x = 6$ or $x = -1$

Tip: The solutions have the **opposite** signs to the numbers in the brackets. $(x - 6)$ gives $x = 6$, not $x = -6$.

- (b) Write down a quadratic equation that has solutions $x = 2$ and $x = -9$

Work backwards.

$$x = 2 \text{ comes from } x - 2 = 0.$$

$$x = -9 \text{ comes from } x + 9 = 0.$$

$$\text{So } (x - 2)(x + 9) = 0$$

$$\text{Expanded, this is } x^2 + 7x - 18 = 0.$$

Answer: $(x - 2)(x + 9) = 0$ (or $x^2 + 7x - 18 = 0$)

2 (3 marks)

(a) Factorise $x^2 + 2x - 35$

Find two numbers that multiply to -35 and add to $+2$.

They are $+7$ and -5 .

Check: $7 \times (-5) = -35$ and $7 + (-5) = 2$ ✓

Answer: $(x + 7)(x - 5)$

Tip: Check your brackets by expanding them. You should get back to $x^2 + 2x - 35$.

Marks: 1 mark for $(x \pm 7)(x \pm 5)$. 1 mark for the correct signs.

(b) Hence solve $x^2 + 2x - 35 = 0$

“Hence” means use part (a): $(x + 7)(x - 5) = 0$

$x + 7 = 0$ gives $x = -7$

$x - 5 = 0$ gives $x = 5$

Answer: $x = -7$ or $x = 5$

3 (3 marks)

Solve $x^2 + 7x + 12 = 0$

Find two numbers that multiply to $+12$ and add to $+7$.

They are $+3$ and $+4$.

So $(x + 3)(x + 4) = 0$

$x + 3 = 0$ gives $x = -3$

$x + 4 = 0$ gives $x = -4$

Answer: $x = -3$ or $x = -4$

Marks: 1 mark for $(x + 3)(x + 4)$. 1 mark for one correct solution. 1 mark for both.

4 (3 marks)

Solve $x^2 - 3x - 10 = 0$

Find two numbers that multiply to -10 and add to -3 .

They are -5 and $+2$.

So $(x - 5)(x + 2) = 0$

$x - 5 = 0$ gives $x = 5$

$x + 2 = 0$ gives $x = -2$

Answer: $x = 5$ or $x = -2$

Tip: Do not stop at $(x - 5)(x + 2)$. The question says “solve”, so you must give the values of x .

Marks: 1 mark for $(x - 5)(x + 2)$. 1 mark for one correct solution. 1 mark for both.

5 (3 marks)

Solve $x^2 - 10x + 21 = 0$

Find two numbers that multiply to $+21$ and add to -10 .

They are -3 and -7 .

So $(x - 3)(x - 7) = 0$

$x - 3 = 0$ gives $x = 3$

$x - 7 = 0$ gives $x = 7$

Answer: $x = 3$ or $x = 7$

Marks: 1 mark for $(x - 3)(x - 7)$. 1 mark for one correct solution. 1 mark for both.

6 (2 marks)

Solve $x^2 + 4x = 0$

Both terms contain x , so take x out as a common factor.

$$x(x + 4) = 0$$

$$\text{So } x = 0 \text{ or } x + 4 = 0$$

$$x = 0 \text{ or } x = -4$$

Answer: $x = 0$ or $x = -4$

Tip: $x = 0$ is a solution too. Do not divide both sides by x , or you will lose it.

Marks: 1 mark for $x(x + 4)$. 1 mark for both solutions.

7 (2 marks)

$$\text{Solve } x^2 - 49 = 0$$

This is a difference of two squares: $x^2 - 7^2$.

$$\text{So } (x + 7)(x - 7) = 0$$

$$x = -7 \text{ or } x = 7$$

(Or: $x^2 = 49$, so $x = 7$ or $x = -7$.)

Answer: $x = 7$ or $x = -7$

Tip: Remember the negative square root: $(-7)^2 = 49$ as well.

Marks: 1 mark for $(x + 7)(x - 7)$ or $x^2 = 49$. 1 mark for both solutions.

8 (3 marks)

$$\text{Solve } x^2 + x - 20 = 0$$

x means $1x$, so the numbers must add to $+1$.

Find two numbers that multiply to -20 and add to $+1$.

They are $+5$ and -4 .

$$\text{So } (x + 5)(x - 4) = 0$$

$$x = -5 \text{ or } x = 4$$

Answer: $x = -5$ or $x = 4$

Marks: 1 mark for $(x + 5)(x - 4)$. 1 mark for one correct solution. 1 mark for both.

9 (3 marks)

Solve $x^2 = 5x + 14$

First rearrange so that one side is 0.

Subtract $5x$ and 14 from both sides: $x^2 - 5x - 14 = 0$

Two numbers that multiply to -14 and add to -5 : -7 and $+2$

So $(x - 7)(x + 2) = 0$

$$x = 7 \text{ or } x = -2$$

Answer: $x = 7$ or $x = -2$

Tip: Rearrange so that one side is 0 **before** you factorise. Take care with the signs when you move $5x$ and 14 .

Marks: 1 mark for $x^2 - 5x - 14 = 0$. 1 mark for $(x - 7)(x + 2)$. 1 mark for both solutions.

10 (1 mark)

Nadia says,

' $x = 0$ is a solution of $x^2 + bx = 0$ for every value of b .'

Is Nadia correct?

You must give a reason for your answer.

Substitute $x = 0$ into $x^2 + bx$: $0^2 + b \times 0 = 0 + 0 = 0$

This is true whatever the value of b .

(Or: $x^2 + bx = x(x + b)$, and $x = 0$ makes the first factor 0.)

So Nadia is correct.

Answer: Yes, because $0^2 + b \times 0 = 0$ for any value of b (or $x^2 + bx = x(x + b) = 0$ gives $x = 0$)

Tip: This is why you must never divide both sides of $x^2 = 6x$ by x : you would lose the solution $x = 0$.

11 (5 marks)

The length of a rectangle is 4 cm more than its width, x cm.

The area of the rectangle is 45 cm^2 .

Work out the length of the rectangle.

The width is x , so the length is $x + 4$.

Area = length $\times$ width, so $x(x + 4) = 45$

Expand the bracket: $x^2 + 4x = 45$

Subtract 45 from both sides: $x^2 + 4x - 45 = 0$

Two numbers that multiply to -45 and add to $+4$: $+9$ and -5

So $(x + 9)(x - 5) = 0$

$x = -9$ or $x = 5$

A width cannot be negative, so $x = 5$.

Length = $5 + 4 = 9 \text{ cm}$

Check: $9 \times 5 = 45$ ✓

Answer: 9 cm

Tip: $x = 5$ is the width. The question asks for the length, so add 4.

Marks: 1 mark for $x(x + 4) = 45$. 1 mark for $x^2 + 4x - 45 = 0$. 1 mark for $(x + 9)(x - 5)$. 1 mark for $x = 5$. 1 mark for 9.
