

Solving Equations Using Graphs

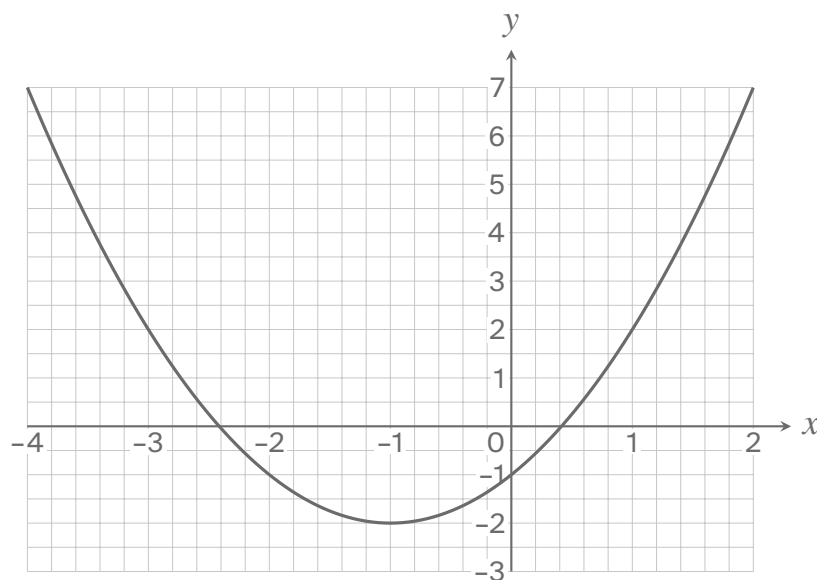
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (6 marks)

Here is the graph of $y = x^2 + 2x - 1$



(a) Use the graph to find estimates for the solutions of the equation $x^2 + 2x - 1 = 0$

The solutions of $x^2 + 2x - 1 = 0$ are where $y = 0$.

That is where the curve crosses the x -axis.

Read off the two x -values: $x \approx -2.4$ and $x \approx 0.4$

Any answers from -2.3 to -2.5 and from 0.3 to 0.5 are fine.

Answer: $x \approx -2.4$ and $x \approx 0.4$

Tip: Write the solutions as x -values. Don't give coordinates like $(0.4, 0)$, and don't round to whole numbers: "estimate" means read the graph as accurately as you can.

Marks: 1 mark for each solution.

- (b) Use the graph to find estimates for the solutions of the equation $x^2 + 2x - 1 = 3$

This time you need the points on the curve where $y = 3$.

Draw a horizontal line at $y = 3$.

It meets the curve twice. Read down to the x -axis:

$$x \approx -3.2 \text{ and } x \approx 1.2$$

Answer: $x \approx -3.2$ and $x \approx 1.2$ (accept -3.1 to -3.3 and 1.1 to 1.3)

Tip: There are two solutions, one on each side of the turning point. Don't forget the negative one.

Marks: 1 mark for using the line $y = 3$. 1 mark for both solutions.

- (c) Write down the coordinates of the turning point of the graph.

The turning point is the lowest point of the curve.

$$\text{It is at } x = -1, \text{ where } y = (-1)^2 + 2 \times (-1) - 1 = 1 - 2 - 1 = -2.$$

Answer: $(-1, -2)$

- (d) Write down the y intercept of the graph of $y = x^2 + 2x - 1$

The y intercept is where the curve crosses the y -axis.

On the y -axis, $x = 0$.

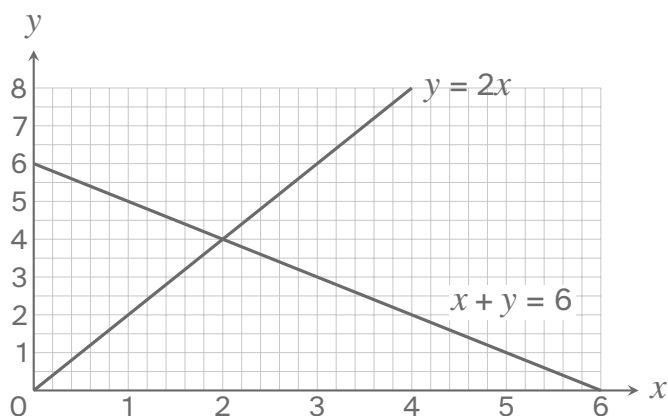
$$y = 0^2 + 2 \times 0 - 1 = -1, \text{ which matches the graph.}$$

Answer: -1 (the point $(0, -1)$)

Tip: The y intercept is on the y -axis (where $x = 0$). Don't give the points where the curve crosses the x -axis.

2 (1 mark)

The diagram shows the graphs of $y = 2x$ and $x + y = 6$



Use these graphs to solve the simultaneous equations

$$y = 2x$$

$$x + y = 6$$

The solution is the point that is on both lines.

The lines cross at (2, 4).

So $x = 2$ and $y = 4$.

Check: $2 \times 2 = 4$ and $2 + 4 = 6$ ✓

Answer: $x = 2$, $y = 4$

Tip: Read the crossing point of the two lines, not where the lines cross the axes. Don't swap x and y .

3 (4 marks)

(a) On the grid, draw the graph of $x + 2y = 8$ for values of x from 0 to 8

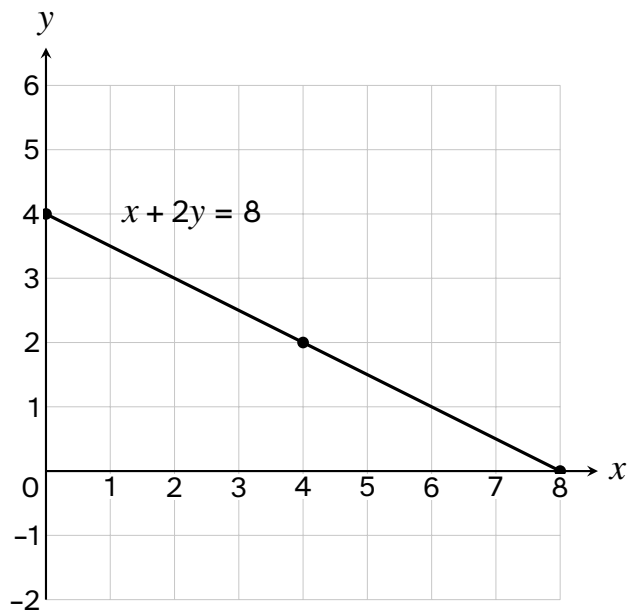
Find some points on the line.

$$x = 0: 2y = 8, \text{ so } y = 4$$

$$x = 4: 4 + 2y = 8, \text{ so } y = 2$$

$$x = 8: 8 + 2y = 8, \text{ so } y = 0$$

Plot (0, 4), (4, 2) and (8, 0) and join them with a ruler.



Tip: For a line like $x + 2y = 8$, put $x = 0$ to find where it crosses the y -axis and $y = 0$ to find where it crosses the x -axis. Then join these two points.

Marks: 2 marks for a correct line from $x = 0$ to $x = 8$. 1 mark for at least 2 correct points.

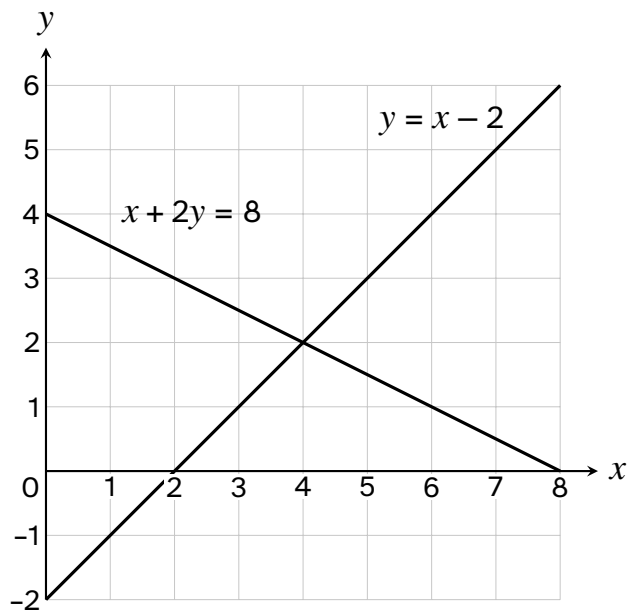
- (b) On the same grid, draw the graph of $y = x - 2$ for values of x from 0 to 8

$$x = 0: y = -2$$

$$x = 4: y = 2$$

$$x = 8: y = 6$$

Plot the points and join them with a ruler.



(c) Use your graphs to solve the simultaneous equations

$$x + 2y = 8$$

$$y = x - 2$$

The lines cross at (4, 2).

So $x = 4$ and $y = 2$.

Check: $4 + 2 \times 2 = 8$ and $4 - 2 = 2$ ✓

Answer: $x = 4$, $y = 2$

4 (1 mark)

Megan says the equation $x^2 + 3 = 1$ has no solutions.

Explain why Megan is correct.

x^2 is never negative, so $x^2 + 3$ is always 3 or more.

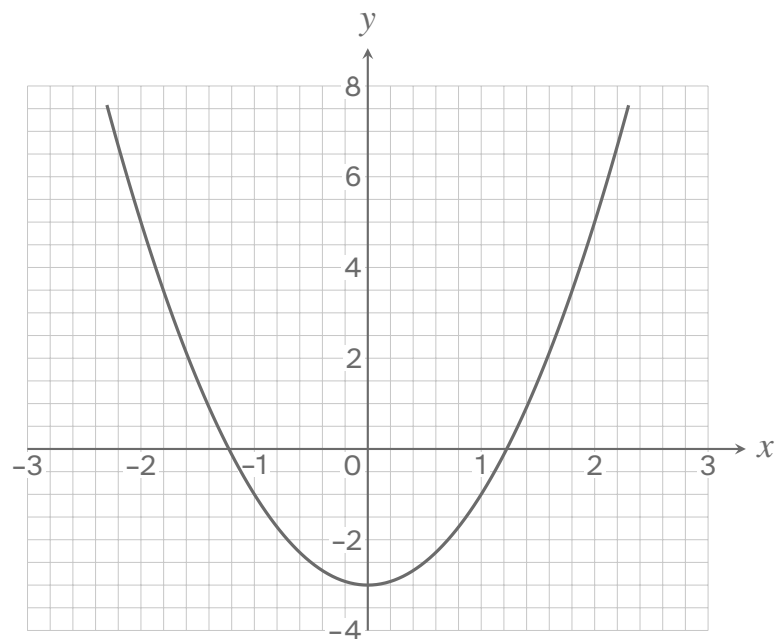
On a graph, $y = x^2 + 3$ has its lowest point at (0, 3).

So it never goes down to $y = 1$, and $x^2 + 3 = 1$ has no solutions.

Answer: $x^2 + 3 \geq 3$ so it can never equal 1

5 (4 marks)

Here is the graph of $y = 2x^2 - 3$



- (a) Use the graph to solve $2x^2 - 3 = 5$

Draw a horizontal line at $y = 5$.

It meets the curve twice. Read down to the x -axis:

$$x = -2 \text{ and } x = 2$$

$$\text{Check: } 2 \times 2^2 - 3 = 8 - 3 = 5 \quad \checkmark$$

Answer: $x = -2$ and $x = 2$

Tip: Check the scale: on this y -axis the labels go up in 2s, so 5 is halfway between 4 and 6.

Marks: 1 mark for each solution.

- (b) Use the graph to find estimates for the solutions of $2x^2 - 3 = 0$

The solutions are where $y = 0$, which is where the curve crosses the x -axis.

$$x \approx -1.2 \text{ and } x \approx 1.2$$

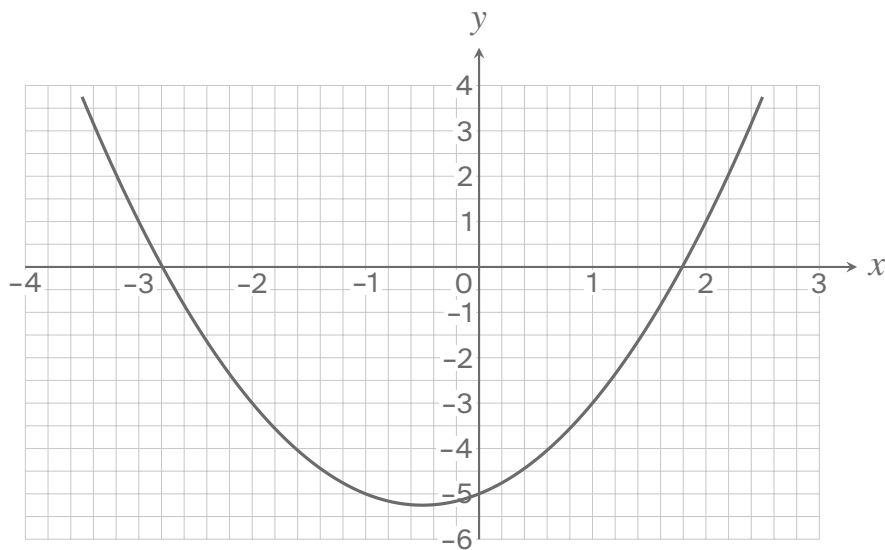
Answers within 0.1 of these are fine.

Answer: $x \approx -1.2$ and $x \approx 1.2$

Marks: 1 mark for each solution.

6 (4 marks)

Here is the graph of $y = x^2 + x - 5$ for values of x from -3.5 to 2.5



- (a) Use the graph to find estimates for the solutions of $x^2 + x - 5 = 0$

Read off where the curve crosses the x -axis:

$x \approx -2.8$ and $x \approx 1.8$

Any answers from -2.7 to -2.9 and from 1.7 to 1.9 are fine.

Answer: $x \approx -2.8$ and $x \approx 1.8$

Tip: The y -intercept (-5) is not a solution. Solutions of “ $= 0$ ” are always on the x -axis.

Marks: 1 mark for each solution.

- (b) Use the graph to find the solutions of $x^2 + x - 5 = -3$

Draw a horizontal line at $y = -3$.

Read down from where it meets the curve: $x = -2$ and $x = 1$

Check: $(-2)^2 + (-2) - 5 = 4 - 2 - 5 = -3$ ✓

Check: $1^2 + 1 - 5 = -3$ ✓

Answer: $x = -2$ and $x = 1$

Marks: 1 mark for using the line $y = -3$. 1 mark for both solutions.
