

Sequences - Term-to-Term Rules and Patterns

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Here are the first four terms of a number sequence.

4 13 22 31

(a) Work out the 8th term of the sequence.

$$13 - 4 = 9, \quad 22 - 13 = 9, \quad 31 - 22 = 9$$

The terms go up by 9 each time.

31, 40, 49, 58, 67

The 8th term is 67.

Answer: 67

(b) Suki says,

‘The 10th term of the sequence is double the 5th term.’

Is Suki correct?

You must show how you get your answer.

$$\text{5th term: } 31 + 9 = 40$$

$$\text{10th term: } 40 + 5 \times 9 = 40 + 45 = 85$$

$$\text{Double the 5th term: } 2 \times 40 = 80$$

85 is not 80, so Suki is not correct.

Answer: No. The 5th term is 40 and the 10th term is 85, which is not $2 \times 40 = 80$

Tip: Doubling the term number does not double the term. Work out both terms and compare them.

Marks: 1 mark for 40 or 85. 1 mark for No with both 40 and 85 (or 80 and 85).

- (c) Which term of the sequence is equal to 94?

Keep adding 9: 67 (8th), 76 (9th), 85 (10th), 94 (11th)

(Or: $94 - 4 = 90$, $90 \div 9 = 10$ steps, so it is the 11th term.)

Answer: the 11th term

Tip: $94 - 4 = 90$ and $90 \div 9 = 10$ gives the number of **steps** from the 1st term, so 94 is the 11th term, not the 10th.

2 (2 marks)

Here are the first four terms of a sequence.

3 6 12 24

- (a) Find the 6th term.

Each term is double the one before.

5th term: $24 \times 2 = 48$

6th term: $48 \times 2 = 96$

Answer: 96

- (b) Write down the term-to-term rule.

$3 \times 2 = 6$, $6 \times 2 = 12$, $12 \times 2 = 24$

Each term is double the previous one.

Answer: multiply by 2

Tip: Write the rule in words, “multiply by 2”. Writing “ $2n$ ” or the next term is not the term-to-term rule.

3 (2 marks)

Here are the first three terms of a sequence.

1 3 7

- (a) Laila says the 4th term is 15
Write down a rule Laila could have used.

Try multiplying by 2 and adding 1: $1 \times 2 + 1 = 3$, $3 \times 2 + 1 = 7$ ✓

Then the 4th term is $7 \times 2 + 1 = 15$ ✓

(Another rule that works: the differences double each time: +2, +4, +8, ... so $7 + 8 = 15$.)

Answer: e.g. multiply by 2 then add 1

- (b) Find a **different** rule that gives a different 4th term.
Write down your rule and the 4th term it gives.

Look at the differences: $3 - 1 = 2$ and $7 - 3 = 4$.

One rule: the differences go up by 2 each time: +2, +4, +6, ...

4th term: $7 + 6 = 13$

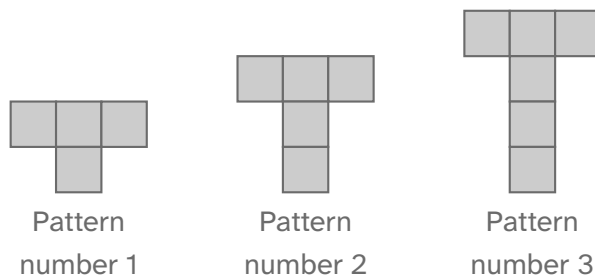
(Other rules are possible, as long as they fit 1, 3, 7 and do not give 15.)

Answer: e.g. add 2, 4, 6, ...; 4th term 13

Tip: Your rule must work for **all** of the terms you are given, not just the first two. Check it on 1, 3 and 7.

4 (4 marks)

Here is a sequence of patterns made from grey tiles.



- (a) Draw pattern number 4.

The top row always has 3 tiles.

The stem gets 1 tile longer each time.

Pattern 4 has 3 tiles across the top and a stem of 4 tiles going down.

Answer: 7 tiles: 3 across and a stem of 4

- (b) Complete the table.

Pattern number	1	2	3	4	5	6
Number of tiles	4	5	6			

Each pattern has 1 more tile than the one before.

$$6 + 1 = 7, 7 + 1 = 8, 8 + 1 = 9$$

Answer: 7, 8, 9

- (c) How many tiles are needed for pattern number 30?

The number of tiles is always 3 more than the pattern number.

So pattern n has $n + 3$ tiles.

$$\text{Pattern 30: } 30 + 3 = 33$$

Answer: 33

- (d) One pattern in this sequence needs twice as many tiles as pattern number 2
Which pattern is it?

Pattern 2 needs $2 + 3 = 5$ tiles.

Twice as many: $2 \times 5 = 10$ tiles

Pattern n needs $n + 3$ tiles, so $n + 3 = 10$

$$n = 7$$

Answer: Pattern number 7

Tip: Pattern number 4 is **not** the answer. Doubling the pattern number does not double the number of tiles.

5 (2 marks)

The first term of a sequence is 2

The term-to-term rule is multiply by 3 then subtract 2

Work out the 5th term of the sequence.

Use the rule: multiply by 3, then subtract 2.

2nd term: $2 \times 3 - 2 = 4$

3rd term: $4 \times 3 - 2 = 10$

4th term: $10 \times 3 - 2 = 28$

5th term: $28 \times 3 - 2 = 82$

Answer: 82

Marks: 1 mark for 10 and 28. 1 mark for 82.

6 (2 marks)

The term-to-term rule of a sequence is multiply by 3 then add 5

The 3rd term of the sequence is 26

Find the 1st term of the sequence.

Work backwards from the 3rd term.

Undo the steps in reverse order: subtract 5 first, then divide by 3.

2nd term: $26 - 5 = 21$, then $21 \div 3 = 7$

1st term: $7 - 5 = 2$, then $2 \div 3 = \frac{2}{3}$

Check: $\frac{2}{3} \times 3 + 5 = 7$ and $7 \times 3 + 5 = 26$ ✓

Answer: $\frac{2}{3}$

Tip: Reverse the **order** as well as the operations. The last step was add 5, so subtract 5 first, then divide by 3.

Marks: 1 mark for 7. 1 mark for $\frac{2}{3}$ (or 0.6).

7 (2 marks)

Here are the first five terms of an arithmetic sequence.

a 11 17 23 b

Find the value of a and the value of b .

In an arithmetic sequence the difference is always the same.

$17 - 11 = 6$, so the difference is 6.

a is one step before 11: $a = 11 - 6 = 5$

b is one step after 23: $b = 23 + 6 = 29$

Check: 5, 11, 17, 23, 29 ✓

Answer: $a = 5$, $b = 29$

Marks: 2 marks for $a = 5$ and $b = 29$. 1 mark for finding the difference of 6, or for one correct value.

8 (2 marks)

Here are the first four terms of a sequence.

5 12 19 26

Which term of the sequence is the first term to be greater than 100?

The terms go up by 7 each time.

Keep adding 7 and count the terms:

5, 12, 19, 26, 33, 40, 47, 54, 61, 68, 75, 82, 89, 96, 103

96 is the 14th term and 103 is the 15th term.

So the 15th term is the first one greater than 100.

Answer: the 15th term (103)

Tip: “Which term” means the position in the sequence (15th), not the value of the term (103).

Marks: 1 mark for continuing the sequence correctly (or using $7n - 2$). 1 mark for 15th.

9 (2 marks)

A sequence of numbers starts at 3

Each number in the sequence is 4 more than the number before it.

(a) Show that 99 is a number in this sequence.

Every term is 3 plus a number of 4s.

$$99 - 3 = 96$$

$$96 \div 4 = 24, \text{ which is a whole number.}$$

So 99 is in the sequence (it is the 25th term).

Answer: $99 - 3 = 96$ and $96 \div 4 = 24$, a whole number, so 99 is a term

(b) Remy says,

‘All the numbers in this sequence are prime numbers.’

Give an example to show that Remy is wrong.

Start at 3 and keep adding 4:

3, 7, 11, 15, ...

15 is in the sequence.

$15 = 3 \times 5$, so 15 is not a prime number.

(Other correct examples include 27 and 35.)

Answer: 15 is in the sequence and $15 = 3 \times 5$ is not prime

Tip: Make your example clear. Don't just list the terms: say which number is not prime and why.
