

Quadratic Graphs

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (5 marks)

(a) Complete the table of values for $y = x^2 - 4x + 1$

x	-1	0	1	2	3	4	5
y	6	1	-2	-3	-2	1	6

Substitute each value of x into $y = x^2 - 4x + 1$.

$$x = 0: 0^2 - 4 \times 0 + 1 = 1$$

$$x = 2: 2^2 - 4 \times 2 + 1 = 4 - 8 + 1 = -3$$

$$x = 4: 4^2 - 4 \times 4 + 1 = 16 - 16 + 1 = 1$$

Check: the table is symmetrical (6, 1, -2, -3, -2, 1, 6) ✓

Answer: 1, -3, 1

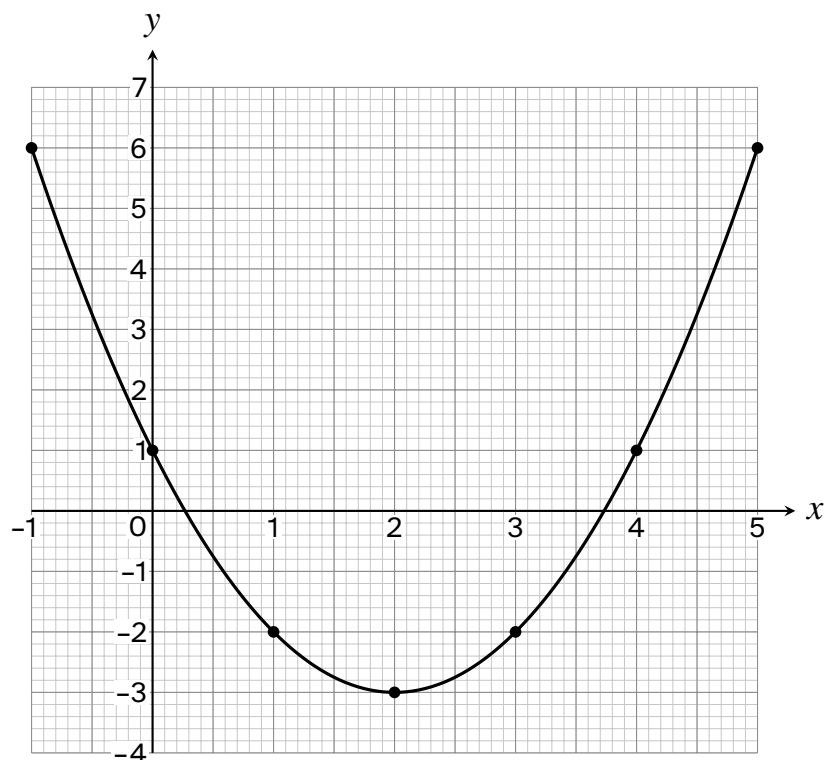
Tip: Work out each term separately, then combine them: at $x = 2$, $x^2 = 4$ and $-4x = -8$, so $y = 4 - 8 + 1 = -3$.

Marks: 2 marks for all three correct. 1 mark for one or two correct.

(b) On the grid, draw the graph of $y = x^2 - 4x + 1$ for values of x from -1 to 5

Plot the seven points from the table.

Join them with one smooth curve.



Tip: Use one smooth curve, not straight lines between the points. The bottom should be rounded, not flat or pointed.

Marks: 1 mark for at least 5 points plotted correctly. 1 mark for a smooth curve.

- (c) Write down the coordinates of the turning point of the graph of $y = x^2 - 4x + 1$

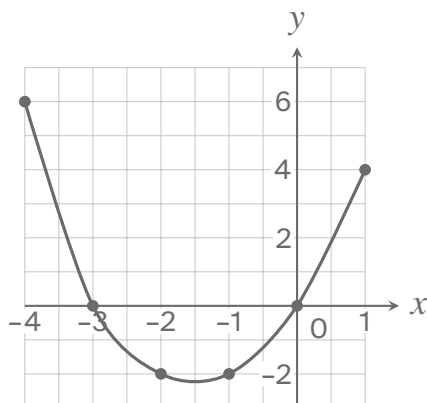
The turning point is the lowest point of the curve.

From the table, it is at $x = 2$, where $y = -3$.

Answer: (2, -3)

2 (1 mark)

Amir draws the graph of $y = x^2 + 3x$ for values of x from -4 to 1
Here is his graph.



Amir has plotted one of the points wrongly.

Find the correct coordinates of the point that Amir has plotted wrongly.

Work out y for each x -value and compare with the graph.

$$x = 1: 1 + 3 = 4 \quad \checkmark \quad x = 0: 0 \quad \checkmark \quad x = -1: 1 - 3 = -2 \quad \checkmark \quad x = -2: 4 - 6 = -2 \quad \checkmark$$

$$x = -3: 9 - 9 = 0 \quad \checkmark$$

$$x = -4: (-4)^2 + 3 \times (-4) = 16 - 12 = 4$$

Amir has plotted $(-4, 6)$, but y should be 4 .

You can also see the mistake from the symmetry: the points at $x = -4$ and $x = 1$ are the same distance from the line of symmetry, so they should be at the same height.

Answer: $(-4, 4)$

Tip: A quadratic graph is symmetrical. If the curve is lopsided, check the points one at a time.

3 (6 marks)

- (a) Complete the table of values for $y = 2x^2 - 4x - 1$

x	-1	0	1	2	3
y	5	-1	-3	-1	5

Substitute each value of x into $y = 2x^2 - 4x - 1$.

$$x = 0: 2 \times 0 - 0 - 1 = -1$$

$$x = 2: 2 \times 2^2 - 4 \times 2 - 1 = 8 - 8 - 1 = -1$$

$$x = 3: 2 \times 3^2 - 4 \times 3 - 1 = 18 - 12 - 1 = 5$$

Answer: -1, -1, 5

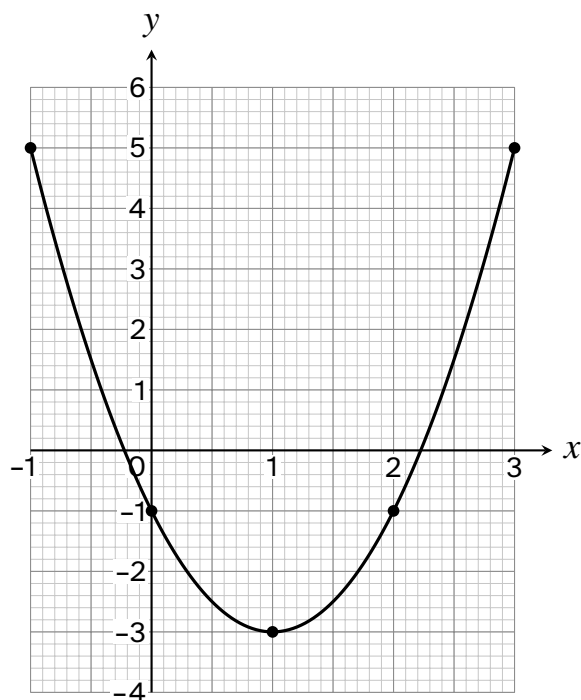
Tip: $2x^2$ means $2 \times x^2$. Square x first, then multiply by 2: at $x = 3$ it is $2 \times 9 = 18$, not $6^2 = 36$.

Marks: 2 marks for all three correct. 1 mark for one or two correct.

- (b) On the grid, draw the graph of $y = 2x^2 - 4x - 1$ for values of x from -1 to 3

Plot the five points and join them with a smooth curve.

The curve is symmetrical about $x = 1$, where it has its lowest point $(1, -3)$.



Tip: Use one smooth U-shaped curve. Do not join the points with straight lines.

Marks: 1 mark for at least 4 points plotted correctly. 1 mark for a smooth curve.

- (c) Use your graph to find estimates of the values of x when $y = 2$

Draw the horizontal line $y = 2$ across the grid.

Read the x -values where the line meets the curve.

$x \approx -0.6$ and $x \approx 2.6$

Any answers from -0.5 to -0.7 and from 2.5 to 2.7 are fine.

Answer: $x \approx -0.6$ and $x \approx 2.6$

Tip: There are two values of x , one on each side of the line of symmetry. Give both.

Marks: 1 mark for each value.

4 (1 mark)

Explain why the graph of $y = x^2 + 4$ does not cross the x -axis.

x^2 is never negative (a square is always 0 or more).

So $x^2 + 4$ is always 4 or more.

So y is never 0, and the graph never reaches the x -axis.

Answer: $x^2 \geq 0$, so $y \geq 4$ for all x .

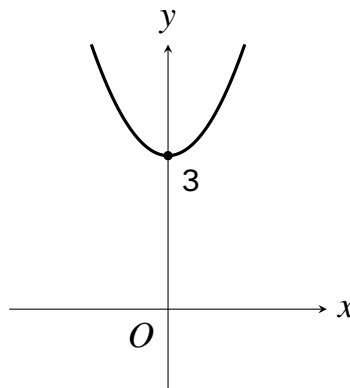
5 (2 marks)

Sketch the graph of $y = x^2 + 3$

$y = x^2 + 3$ is the U-shaped graph of $y = x^2$, moved up by 3.

When $x = 0$, $y = 3$, so the lowest point is $(0, 3)$ on the y -axis.

The curve is symmetrical about the y -axis.

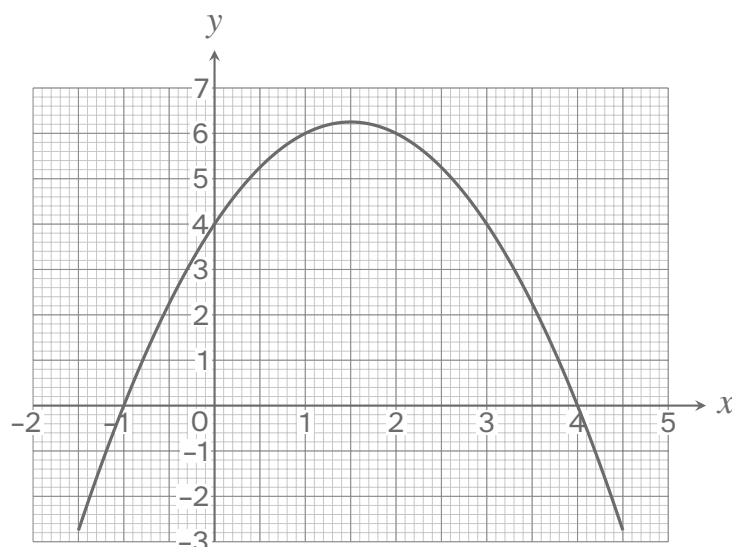


Tip: Label where the curve meets the y -axis with a 3. A sketch without the key value labelled can lose a mark.

Marks: 1 mark for a U-shaped curve. 1 mark for the turning point at $(0, 3)$ on the y -axis, labelled.

6 (5 marks)

Here is the graph of $y = 4 + 3x - x^2$



- (a) Write down the coordinates of the point where the graph crosses the y -axis.

The graph crosses the y -axis where $x = 0$.

$$y = 4 + 3 \times 0 - 0^2 = 4$$

Answer: (0, 4)

- (b) Use the graph to solve $4 + 3x - x^2 = 0$

The solutions are where $y = 0$, which is where the curve crosses the x -axis.

The curve crosses at $x = -1$ and $x = 4$.

$$\text{Check: } 4 + 3 \times 4 - 4^2 = 4 + 12 - 16 = 0 \quad \checkmark$$

Answer: $x = -1$ and $x = 4$

Tip: The solutions are x -values on the x -axis. Don't give the y -intercept or the highest point.

Marks: 1 mark for each solution.

- (c) The graph is symmetrical.
Write down the equation of its line of symmetry.

The line of symmetry is halfway between the solutions -1 and 4 .

$$(-1 + 4) \div 2 = 1.5$$

It is a vertical line, so its equation is $x = 1.5$

Answer: $x = 1.5$

Tip: A vertical line has an equation of the form $x = \dots$

- (d) Use the graph to find the maximum value of y .

The maximum is the highest point of the curve, on the line of symmetry $x = 1.5$

Reading from the graph, y is about 6.2

Exactly: $4 + 3 \times 1.5 - 1.5^2 = 4 + 4.5 - 2.25 = 6.25$

Answer: 6.25 (accept 6.2 to 6.3)

7 (6 marks)

A ball is thrown upwards.

Its height, h metres, after t seconds is given by $h = 20t - 5t^2$

- (a) Complete the table of values.

t	0	1	2	3	4
h	0	15	20	15	0

Substitute each value of t into $h = 20t - 5t^2$.

$$t = 2: 20 \times 2 - 5 \times 2^2 = 40 - 20 = 20$$

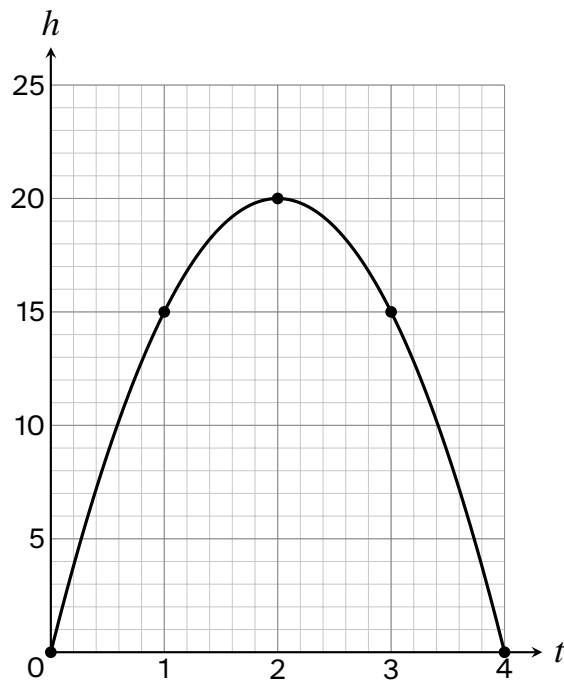
$$t = 4: 20 \times 4 - 5 \times 4^2 = 80 - 80 = 0$$

Answer: 20, 0

- (b) On the grid, draw the graph of h against t .

Plot the five points from the table.

Join them with a smooth $\cap$ -shaped curve.



Marks: 1 mark for at least 4 points plotted correctly. 1 mark for a smooth curve.

- (c) Write down the greatest height of the ball.

The greatest height is the top of the curve.

This is at $t = 2$, where $h = 20$.

Answer: 20 m

- (d) Use your graph to find the two times when the ball is 10 m above the ground.

Draw a horizontal line across from $h = 10$.

It meets the curve twice: once on the way up and once on the way down.

Read down to the t -axis: $t \approx 0.6$ and $t \approx 3.4$

Answer: $t \approx 0.6$ and $t \approx 3.4$

Marks: 1 mark for each time (read from your graph).