

Prime Factorisation, HCF and LCM

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Here is an incomplete factor tree for 150

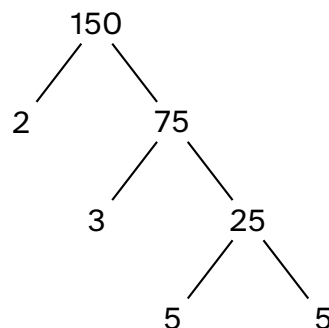
(a) Complete the factor tree for 150

$150 = 2 \times 75$, so the first gap is 75.

$75 = 3 \times 25$, so the next gap is 25.

25 is not prime, so split it: $25 = 5 \times 5$

5 is prime, so the tree is finished.



Answer: 75, 25, 5, 5

Marks: 2 marks for all four correct. 1 mark for two correct.

(b) Write 150 as a product of its prime factors.

Multiply together the primes at the ends of the branches: 2, 3, 5 and 5.

$$150 = 2 \times 3 \times 5 \times 5$$

Using a power for the two 5s: $2 \times 3 \times 5^2$

Answer: $2 \times 3 \times 5^2$

Tip: “Product” means multiply. Write the primes with $\times$ signs, not as a list with commas or with + signs.

2 (2 marks)

Write 204 as a product of its prime factors.

Keep dividing by prime numbers, starting with 2:

$$204 = 2 \times 102$$

$$102 = 2 \times 51$$

$$51 = 3 \times 17$$

17 is prime, so stop.

$$204 = 2 \times 2 \times 3 \times 17 = 2^2 \times 3 \times 17$$

Answer: $2^2 \times 3 \times 17$

Tip: 51 looks prime but is not: $51 = 3 \times 17$. Check each number by dividing by 2, 3, 5, 7, ...

Marks: 1 mark for a correct method, e.g. a factor tree with at least two correct steps. 1 mark for $2^2 \times 3 \times 17$ (or $2 \times 2 \times 3 \times 17$).

3 (2 marks)

Write 392 as a product of powers of its prime factors.

$$392 = 2 \times 196$$

$$196 = 2 \times 98$$

$$98 = 2 \times 49$$

$$49 = 7 \times 7$$

$$\text{So } 392 = 2 \times 2 \times 2 \times 7 \times 7$$

“Powers” means write repeated primes as powers: $2^3 \times 7^2$

Answer: $2^3 \times 7^2$

Tip: “Product of powers” means you must use index form: $2^3 \times 7^2$, not $2 \times 2 \times 2 \times 7 \times 7$.

Marks: 1 mark for a correct method, e.g. $2 \times 2 \times 2 \times 7 \times 7$. 1 mark for $2^3 \times 7^2$.

4 (2 marks)

Find the Highest Common Factor (HCF) of 54 and 126

Write each number as a product of primes:

$$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$$

$$126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$$

Take the primes that are in **both**: one 2 and two 3s.

$$\text{HCF} = 2 \times 3 \times 3 = 18$$

$$\text{Check: } 54 \div 18 = 3 \text{ and } 126 \div 18 = 7 \quad \checkmark$$

Answer: 18

Tip: A common factor such as 2, 6 or 9 is not enough: the question asks for the **highest** one.

Marks: 1 mark for the prime factors of both numbers, or a list of factors of both. 1 mark for 18.

5 (2 marks)

Find the Lowest Common Multiple (LCM) of 18 and 30

List the multiples of 30: 30, 60, 90, ...

Check each one: is it in the 18 times table?

30 and 60 are not, but $90 = 18 \times 5$ is.

Or use primes: $18 = 2 \times 3^2$ and $30 = 2 \times 3 \times 5$

$$\text{LCM} = 2 \times 3^2 \times 5 = 90$$

Answer: 90

Tip: The LCM is a multiple of both numbers, so it cannot be smaller than 30. If you get 6, you have found the HCF.

Marks: 1 mark for listing multiples of both numbers, or for their prime factors. 1 mark for 90.

6 (2 marks)

$$A = 2^3 \times 3^2 \times 5 \qquad B = 3^3 \times 5$$

(a) Find the HCF of A and B .

For the HCF, use only the primes that are in both A and B .

2 is only in A , so leave it out.

3 is in both: the lower power is 3^2 .

5 is in both: the lower power is 5.

$$\text{HCF} = 3^2 \times 5 = 9 \times 5 = 45$$

Answer: 45

(b) Find the LCM of A and B .

For the LCM, use every prime that appears, with the higher power.

2: 2^3

3: 3^3

5: 5

$$\text{LCM} = 2^3 \times 3^3 \times 5 = 8 \times 27 \times 5 = 1080$$

Answer: 1080

7 (3 marks)

Paper plates are sold in packs of 20

Paper cups are sold in packs of 12

Rhys is buying plates and cups for a party.

He wants to have equal numbers of plates and cups.

He needs at least 100 plates and at least 100 cups.

Work out the least number of packs of plates and the least number of packs of cups he can buy.

The number of plates must be a multiple of 20, and the number of cups a multiple of 12.

Equal numbers of each means a common multiple of 20 and 12.

Multiples of 20: 20, 40, 60, 80, 100, 120, ...

Multiples of 12: 12, 24, 36, 48, 60, ..., 108, 120, ...

The LCM is 60, so the common multiples are 60, 120, 180, ...

60 is less than 100, so Rhys needs 120 of each.

Plates: $120 \div 20 = 6$ packs

Cups: $120 \div 12 = 10$ packs

Answer: 6 packs of plates and 10 packs of cups

Tip: 120 is the number of plates and cups, not the number of packs. Divide by the pack size to find the number of packs.

Marks: 1 mark for 60 (the LCM of 20 and 12). 1 mark for 120. 1 mark for 6 packs of plates and 10 packs of cups.

8 (3 marks)

In a park, sprinkler A turns on every 40 minutes.

Sprinkler B turns on every 50 minutes.

Both sprinklers turn on together at 6 am.

Work out the first time after 12 noon that both sprinklers turn on together.

They turn on together after a number of minutes that is a multiple of 40 **and** of 50.

So find the LCM of 40 and 50.

Multiples of 50: 50, 100, 150, 200, ...

200 is the first one in the 40 times table: $40 \times 5 = 200$

200 minutes = 3 hours 20 minutes.

Together at: 6:00 am $\rightarrow$ 9:20 am $\rightarrow$ 12:40 pm

9:20 am is before noon, so the answer is 12:40 pm.

Answer: 12:40 pm

Tip: 200 minutes is 3 hours 20 minutes, not 2 hours: divide by 60 to change minutes to hours.

Marks: 1 mark for listing multiples of 40 and 50 (or prime factors). 1 mark for 200 minutes. 1 mark for 12:40 pm.

9 (3 marks)

A food bank has 60 tins of soup and 96 packets of pasta.

It puts all of them into food parcels.

Each parcel has the same number of tins and the same number of packets.

(a) What is the greatest number of parcels it can make?

The number of parcels must divide exactly into 60 **and** into 96.

So find the HCF of 60 and 96.

$$60 = 2^2 \times 3 \times 5$$

$$96 = 2^5 \times 3$$

Primes in both, lower powers: $2^2 \times 3$

$$\text{HCF} = 4 \times 3 = 12 \text{ parcels}$$

Answer: 12

Tip: Sharing into the greatest number of equal groups is an HCF question. The answer must divide into both 60 and 96, so it cannot be 480 (the LCM).

Marks: 1 mark for a method to find the HCF, e.g. lists of factors or prime factors. 1 mark for 12.

(b) How many tins and how many packets are in each parcel?

Tins: $60 \div 12 = 5$

Packets: $96 \div 12 = 8$

Answer: 5 tins and 8 packets

10 (1 mark)

Sara says the highest common factor of 12 and 18 is 36

Explain why Sara is wrong.

A factor of 12 must divide into 12, so it cannot be bigger than 12.

36 is a multiple of 12 and 18. In fact 36 is their LCM.

The HCF is 6, the highest number that divides into both 12 and 18.

Answer: 36 is the LCM; the HCF is 6

11 (1 mark)

$$168 = 2^n \times 3 \times 7$$

Find the value of n .

$$3 \times 7 = 21$$

$$168 \div 21 = 8$$

$$\text{So } 2^n = 8$$

$$2 \times 2 \times 2 = 8, \text{ so } 2^3 = 8 \text{ and } n = 3$$

Answer: 3
