

nth Term of Linear Sequences

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Here are the first five terms of an arithmetic sequence.

5 9 13 17 21

Find an expression, in terms of n , for the n th term of this sequence.

The terms go up by 4 each time (the common difference).

So the n th term starts with $4n$.

Write out the $4n$ sequence: 4, 8, 12, 16

Each term of our sequence is 1 more than this.

So the n th term is $4n + 1$.

Check with $n = 2$: $4 \times 2 + 1 = 9$ ✓

Answer: $4n + 1$

Tip: Don't stop at $4n$, and don't write $n + 4$. The difference goes in front of n , then you need the right number on the end.

Marks: 2 marks for $4n + 1$. 1 mark for $4n + k$ (the number in front of n correct).

2 (2 marks)

Here are the first five terms of an arithmetic sequence.

2 8 14 20 26

Find an expression, in terms of n , for the n th term of this sequence.

The terms go up by 6 each time (the common difference).

So the n th term starts with $6n$.

Write out the $6n$ sequence: 6, 12, 18, 24

Each term of our sequence is 4 less than this.

So the n th term is $6n - 4$.

Check with $n = 2$: $6 \times 2 - 4 = 8$ ✓

Answer: $6n - 4$

Marks: 2 marks for $6n - 4$. 1 mark for $6n + k$ (the number in front of n correct).

3 (2 marks)

Here are the first five terms of an arithmetic sequence.

30 27 24 21 18

Find an expression, in terms of n , for the n th term of this sequence.

The terms go down by 3 each time, so the common difference is -3 .

So the n th term starts with $-3n$.

Write out the $-3n$ sequence: -3 , -6 , -9 , -12

Each term of our sequence is 33 more than this.

So the n th term is $-3n + 33$, which is the same as $33 - 3n$.

Check with $n = 2$: $-3 \times 2 + 33 = 27$ ✓

Answer: $33 - 3n$

Tip: A decreasing sequence has a negative number in front of n . Check your answer by putting $n = 1$ into it: you should get 30.

Marks: 2 marks for $33 - 3n$. 1 mark for $-3n + k$ (the number in front of n correct).

4 (3 marks)

Here are the first five terms of an arithmetic sequence.

4 11 18 25 32

(a) Find the 12th term of the sequence.

The terms go up by 7, so the n th term starts with $7n$.

$7n$ gives 7, 14, 21, ..., and each term is 3 less than this.

So the n th term is $7n - 3$.

12th term: $7 \times 12 - 3 = 84 - 3 = 81$

Answer: 81

- (b) Find the first term of the sequence that is greater than 150

The n th term is $7n - 3$.

We need $7n - 3 > 150$.

Add 3: $7n > 153$

Divide by 7: $n > 21.8 \dots$

So try $n = 21$ and $n = 22$.

$n = 21$: $7 \times 21 - 3 = 144$ (not greater than 150)

$n = 22$: $7 \times 22 - 3 = 151$

Answer: 151

Tip: The question asks for the **term** (151), not its position in the sequence ($n = 22$).

Marks: 1 mark for $7n - 3 > 150$ or finding terms close to 150. 1 mark for 151.

5 (5 marks)

Here are the first three patterns in a sequence made from sticks.



Pattern
number 1



Pattern
number 2



Pattern
number 3

- (a) Write down an expression, in terms of n , for the number of sticks in pattern number n .

Count the sticks: 7, 10, 13, ...

They go up by 3 each time, so the n th term starts with $3n$.

$3n$ gives 3, 6, 9, and each term is 4 more than this.

So the expression is $3n + 4$.

Answer: $3n + 4$

Marks: 2 marks for $3n + 4$. 1 mark for $3n + k$.

- (b) Which pattern number uses 64 sticks?

Solve $3n + 4 = 64$.

Subtract 4: $3n = 60$

Divide by 3: $n = 20$

So it is pattern number 20.

Answer: 20

- (c) Kasia has 100 sticks.
She wants to make pattern number 40
How many more sticks does Kasia need?

Pattern number 40 uses $3 \times 40 + 4 = 120 + 4 = 124$ sticks.

Kasia has 100 sticks.

$124 - 100 = 24$

Answer: 24

Tip: Don't work out $3 \times 40 = 120$ and forget the +4. Use the whole expression from part (a).

Marks: 1 mark for 124 sticks in pattern 40. 1 mark for 24.

6 (2 marks)

Here are the first four terms of an arithmetic sequence.

5 8 11 14

Find the 20th term of the sequence.

The terms go up by 3, so the n th term starts with $3n$.

$3n$ gives 3, 6, 9, 12, and each term is 2 more than this.

So the n th term is $3n + 2$.

20th term: $3 \times 20 + 2 = 62$

Answer: 62

Marks: 1 mark for $3n + 2$ or another correct method. 1 mark for 62.

7 (3 marks)

The 3rd term of an arithmetic sequence is 18

The 7th term of the sequence is 38

Find the n th term of the sequence.

From the 3rd term to the 7th term is 4 steps.

The terms go up by $38 - 18 = 20$ in those 4 steps.

So the common difference is $20 \div 4 = 5$.

The n th term starts with $5n$.

For the 3rd term: $5 \times 3 = 15$, but the 3rd term is 18.

$18 - 15 = 3$, so add 3.

The n th term is $5n + 3$.

Check the 7th term: $5 \times 7 + 3 = 38$ ✓

Answer: $5n + 3$

Tip: Count the steps between the terms, not the terms. From the 3rd to the 7th term is $7 - 3 = 4$ steps.

Marks: 1 mark for the difference of 5. 1 mark for a correct method to find the number on the end. 1 mark for $5n + 3$.

8 (1 mark)

The n th term of a sequence is $2n + 6$

Hamid says 51 is a term in the sequence.

Explain why Hamid is wrong.

$2n$ is always even.

Even + 6 is always even.

So every term of $2n + 6$ is even.

51 is odd, so it cannot be a term.

Answer: $2n + 6$ is always even, so 51 cannot be a term.

9 (3 marks)

Here are the first four terms of an arithmetic sequence.

4 13 22 31

How many terms of the sequence are between 100 and 200?

The terms go up by 9, and $9n$ gives 9, 18, 27, ...

Each term is 5 less, so the n th term is $9n - 5$.

First term above 100: $9n - 5 > 100$ gives $n > 11.6 \dots$, so $n = 12$ (the 12th term is 103).

Last term below 200: $9n - 5 < 200$ gives $n < 22.7 \dots$, so $n = 22$ (the 22nd term is 193).

The terms between 100 and 200 are the 12th to the 22nd terms.

Number of terms: $22 - 12 + 1 = 11$

Answer: 11

Tip: From the 12th to the 22nd term there are $22 - 12 + 1 = 11$ terms, not 10. Remember to count both ends.

Marks: 1 mark for $9n - 5$. 1 mark for finding the 12th term (103) or the 22nd term (193). 1 mark for 11.

10 (2 marks)

Here are the first four terms of an arithmetic sequence.

11 8 5 2

Aisha says,

“The 20th term of this sequence is -49 ”

Is Aisha correct?

You must show how you get your answer.

The terms go down by 3, so the n th term starts with $-3n$.

$-3n$ gives $-3, -6, -9, \dots$, and each term is 14 more than this.

So the n th term is $14 - 3n$.

20th term: $14 - 3 \times 20 = 14 - 60 = -46$

-46 is not -49 , so Aisha is not correct.

(Aisha may have worked out $11 - 3 \times 20$, using the 1st term instead of 14.)

Answer: No, the 20th term is -46

Tip: The number on the end of the n th term is not the 1st term. Check your formula with $n = 1$:
 $14 - 3 = 11$ ✓

Marks: 1 mark for $14 - 3n$ or another correct method to find the 20th term. 1 mark for No with -46 .
