

Identities and Algebraic Arguments

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Tick the statement that is an identity.

$$3x + 1 = 10$$

$$x^2 - 9 \equiv (x - 3)(x + 3)$$

$$y = 2x + 5$$

$$4x > 12$$

Tick

✓

An identity is true for every value of x . It uses the sign $\equiv$.

Expanding $(x - 3)(x + 3)$ always gives $x^2 - 9$.

$3x + 1 = 10$ is an equation, $y = 2x + 5$ is a formula, and $4x > 12$ is an inequality.

Tip: Learn the difference: an equation is true for some values, an identity for all values, a formula links different quantities, and an inequality uses $<$ or $>$.

2 (2 marks)

Show that $5(x + 3) - 2(x + 4) \equiv 3x + 7$

Expand the first bracket: $5(x + 3) = 5x + 15$

Expand the second bracket: $-2(x + 4) = -2x - 8$

So the left side is $5x + 15 - 2x - 8$

Collect like terms: $3x + 7$, as required.

Tip: The minus sign multiplies both terms: $-2(x + 4) = -2x - 8$, not $-2x + 8$.

Marks: 1 mark for $5x + 15$ or $-2x - 8$. 1 mark for a complete method.

3 (2 marks)

Show that $(x + 4)(x - 1) - 3x \equiv x^2 - 4$

Expand the double bracket: $(x + 4)(x - 1) = x^2 - x + 4x - 4$

Simplify: $x^2 + 3x - 4$

Subtract $3x$: $x^2 + 3x - 4 - 3x$

$= x^2 - 4$, as required.

Marks: 1 mark for $x^2 + 3x - 4$. 1 mark for completing the working to get $x^2 - 4$.

4 (3 marks)

a and b are integers.

$$3(2x + a) - b(x - 1) \equiv 4x + 17$$

Find the value of a and the value of b .

Expand both brackets:

$$3(2x + a) = 6x + 3a$$

$$-b(x - 1) = -bx + b$$

$$\text{So } 6x - bx + 3a + b \equiv 4x + 17$$

Compare the x -terms: $6 - b = 4$, so $b = 2$

Compare the numbers: $3a + b = 17$

With $b = 2$: $3a = 15$, so $a = 5$

Answer: $a = 5$, $b = 2$

Tip: Take care with the minus sign: $-b \times (-1) = +b$. Then match the x -terms on each side, and match the numbers.

Marks: 1 mark for expanding the brackets. 1 mark for $b = 2$. 1 mark for $a = 5$.

5 (2 marks)

$$2x + 5 \qquad 4x - 1 \qquad 3x - 1$$

Show that the mean of these three expressions is $3x + 1$

Mean = total $\div$ number of expressions

$$\text{Total: } (2x + 5) + (4x - 1) + (3x - 1) = 9x + 3$$

$$\text{Divide by 3: } \frac{9x + 3}{3}$$

$$9x \div 3 = 3x \text{ and } 3 \div 3 = 1$$

So the mean is $3x + 1$, as required.

Marks: 1 mark for $9x + 3$. 1 mark for dividing by 3 to get $3x + 1$.

6 (1 mark)

n is a positive integer.

Ria says,

' $4n - 2$ is always an odd number.'

Explain why Ria is wrong.

$4n$ is a multiple of 4, so it is even.

2 is even.

Even $-$ even = even.

So $4n - 2$ is always even, never odd.

(Or: $4n - 2 = 2(2n - 1)$, which is 2 times a whole number.)

Answer: $4n - 2 = 2(2n - 1)$ is always even.

7 (4 marks)

n is a whole number, so $2n + 1$ is an odd number.

- (a) Write down, in terms of n , the next odd number after $2n + 1$

Odd numbers go up in 2s.

$$2n + 1 + 2 = 2n + 3$$

Answer: $2n + 3$

- (b) Shreya says,

‘If you add any four consecutive odd numbers, the answer is always a multiple of 8’

Show that Shreya is correct.

Four consecutive odd numbers are $2n + 1$, $2n + 3$, $2n + 5$ and $2n + 7$

$$\text{Sum} = (2n + 1) + (2n + 3) + (2n + 5) + (2n + 7)$$

$$= 8n + 16$$

$$= 8(n + 2)$$

$n + 2$ is a whole number, so $8(n + 2)$ is always a multiple of 8. Shreya is correct.

Tip: Examples such as $1 + 3 + 5 + 7 = 16$ do not show it is **always** true. Use algebra.

Marks: 1 mark for four consecutive odd numbers in terms of n . 1 mark for $8n + 16$. 1 mark for $8(n + 2)$ with a conclusion.

8 (3 marks)

Show that the sum of any three consecutive integers is always a multiple of 3

Consecutive integers follow on from each other.

Let the integers be n , $n + 1$ and $n + 2$.

Sum: $n + (n + 1) + (n + 2) = 3n + 3$

Factorise: $3n + 3 = 3(n + 1)$

This is 3 times a whole number, so it is always a multiple of 3.

Tip: Finish with a sentence that says why your expression is a multiple of 3. Leaving the answer as $3n + 3$ can lose the last mark.

Marks: 1 mark for n , $n + 1$, $n + 2$. 1 mark for $3n + 3$. 1 mark for $3(n + 1)$ with a conclusion.

9 (3 marks)

n is an integer.

Show that $(n + 2)(n + 6) - n(n + 4)$ is always a multiple of 4

$$(n + 2)(n + 6) = n^2 + 6n + 2n + 12 = n^2 + 8n + 12$$

$$n(n + 4) = n^2 + 4n$$

Subtract: $(n^2 + 8n + 12) - (n^2 + 4n)$

$$= n^2 + 8n + 12 - n^2 - 4n$$

$$= 4n + 12$$

$$= 4(n + 3)$$

$n + 3$ is an integer, so $4(n + 3)$ is always a multiple of 4.

Tip: Put the second expansion in a bracket before you subtract it, so that every sign changes.

Marks: 1 mark for one correct expansion. 1 mark for $4n + 12$. 1 mark for $4(n + 3)$ with a conclusion.

10 (4 marks)

The diagram shows a rectangle.

All measurements are in centimetres.

The perimeter of the rectangle is P cm.

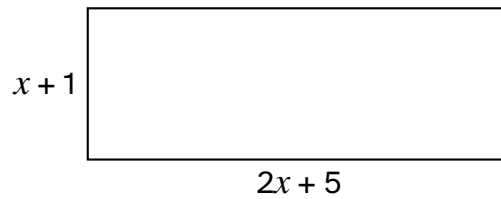


Diagram **NOT** accurately drawn

- (a) Show that $P = 6x + 12$

The perimeter is the total of all four sides.

$$P = (2x + 5) + (x + 1) + (2x + 5) + (x + 1)$$

$$P = 2(2x + 5) + 2(x + 1)$$

$$P = 4x + 10 + 2x + 2$$

$$P = 6x + 12, \text{ as required.}$$

Marks: 1 mark for adding the four sides, e.g. $2(2x + 5) + 2(x + 1)$. 1 mark for $6x + 12$ from correct working.

- (b) Jamal says,

‘When x is a whole number, P is always a multiple of 12’

Is Jamal correct?

You must give a reason for your answer.

$$P = 6x + 12 = 6(x + 2), \text{ so } P \text{ is always a multiple of 6.}$$

It is only a multiple of 12 when $x + 2$ is even.

$$\text{Try } x = 1: P = 6 \times 1 + 12 = 18$$

18 is not a multiple of 12, so Jamal is wrong.

Answer: No. For example, $x = 1$ gives $P = 18$, which is not a multiple of 12.

Tip: One counter-example is enough to show that a statement is wrong. Show the value of x you used and the value of P it gives.

Marks: 1 mark for a value of x that gives P not a multiple of 12, or for $6(x + 2)$. 1 mark for No with a correct reason.
