

Factorising Quadratics

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Factorise $x^2 + 5x + 4$

Find two numbers that multiply to make 4 and add to make 5.

Factor pairs of 4: 1×4 , 2×2

$$1 + 4 = 5 \quad \checkmark$$

So the numbers are 1 and 4.

$$\text{Check by expanding: } (x + 1)(x + 4) = x^2 + 4x + x + 4 = x^2 + 5x + 4 \quad \checkmark$$

Answer: $(x + 1)(x + 4)$

Tip: List the factor pairs of the number term, then pick the pair that adds to the x coefficient.

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 4 or add to 5. 1 mark for $(x + 1)(x + 4)$.

2 (2 marks)

Factorise $x^2 + 8x + 16$

Find two numbers that multiply to make 16 and add to make 8.

$$4 \times 4 = 16 \quad \text{and} \quad 4 + 4 = 8 \quad \checkmark$$

So $(x + 4)(x + 4)$, which can be written as $(x + 4)^2$.

Answer: $(x + 4)^2$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 16 or add to 8. 1 mark for $(x + 4)^2$.

3 (2 marks)

Factorise $x^2 - 4x - 21$

Find two numbers that multiply to make -21 and add to make -4 .

The product is negative, so one number is positive and one is negative.

$$-7 \times 3 = -21 \text{ and } -7 + 3 = -4 \quad \checkmark$$

So the numbers are -7 and $+3$.

Answer: $(x - 7)(x + 3)$

Tip: Check the signs by expanding. $(x + 7)(x - 3)$ gives $+4x$, not $-4x$.

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to -21 or add to -4 .
1 mark for $(x - 7)(x + 3)$.

4 (2 marks)

Factorise $x^2 + 11x + 18$

Find two numbers that multiply to make 18 and add to make 11.

Factor pairs of 18: 1×18 , 2×9 , 3×6

$$2 + 9 = 11 \quad \checkmark$$

Answer: $(x + 2)(x + 9)$

Tip: The answer must be two brackets. $x(x + 11) + 18$ is not factorised.

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 18 or add to 11. 1 mark for $(x + 2)(x + 9)$.

5 (2 marks)

Factorise

(a) $x^2 - 81$

$$x^2 - 81 = x^2 - 9^2$$

This is the difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

$$\text{So } x^2 - 9^2 = (x + 9)(x - 9)$$

Answer: $(x + 9)(x - 9)$

Tip: There is no x term, so the two numbers must add to 0: +9 and -9.

(b) $y^2 - 1$

$$1 = 1^2, \text{ so } y^2 - 1 = y^2 - 1^2$$

This is the difference of two squares.

$$\text{So } y^2 - 1^2 = (y + 1)(y - 1)$$

Answer: $(y + 1)(y - 1)$

6 (2 marks)

Factorise $x^2 - 7x + 12$

Find two numbers that multiply to make +12 and add to make -7.

The product is positive but the sum is negative, so both numbers are negative.

$$(-3) \times (-4) = 12 \text{ and } -3 + (-4) = -7 \checkmark$$

Answer: $(x - 3)(x - 4)$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to 12 or add to -7 . 1 mark for $(x - 3)(x - 4)$.

7 (2 marks)

Factorise $x^2 + 2x - 35$

Find two numbers that multiply to make -35 and add to make $+2$.

One number is positive and one is negative.

$$7 \times (-5) = -35 \text{ and } 7 + (-5) = 2 \checkmark$$

So the numbers are $+7$ and -5 .

Answer: $(x + 7)(x - 5)$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to -35 or add to 2. 1 mark for $(x + 7)(x - 5)$.

8 (4 marks)

(a) Expand and simplify $(x + 3)(x - 8)$

$$x \times x = x^2$$

$$x \times (-8) = -8x$$

$$3 \times x = 3x$$

$$3 \times (-8) = -24$$

$$\text{So } (x + 3)(x - 8) = x^2 - 8x + 3x - 24$$

$$\text{Collect the } x \text{ terms: } -8x + 3x = -5x$$

Answer: $x^2 - 5x - 24$

Marks: 1 mark for four terms with at least three correct. 1 mark for $x^2 - 5x - 24$.

- (b) Factorise $x^2 - 2x - 24$

Find two numbers that multiply to make -24 and add to make -2 .

$$-6 \times 4 = -24 \text{ and } -6 + 4 = -2 \quad \checkmark$$

So the numbers are -6 and $+4$.

Answer: $(x - 6)(x + 4)$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to -24 or add to -2 . 1 mark for $(x - 6)(x + 4)$.

9 (3 marks)

- (a) Factorise $x^2 - 3x$

There is no number term, so take out the common factor.

Both terms have a common factor of x .

$$x^2 \div x = x \text{ and } -3x \div x = -3$$

Answer: $x(x - 3)$

- (b) Factorise $x^2 - 3x - 10$

Find two numbers that multiply to make -10 and add to make -3 .

$$-5 \times 2 = -10 \text{ and } -5 + 2 = -3 \quad \checkmark$$

So the numbers are -5 and $+2$.

Answer: $(x - 5)(x + 2)$

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to -10 or add to -3 . 1 mark for $(x - 5)(x + 2)$.

10 (3 marks)

The area of a square is $(x^2 + 12x + 36) \text{ cm}^2$

- (a) Find an expression, in terms of x , for the length of one side of the square.

Area of a square = side $\times$ side

Factorise the area: find two numbers that multiply to 36 and add to 12.

$$6 \times 6 = 36 \text{ and } 6 + 6 = 12$$

$$\text{So } x^2 + 12x + 36 = (x + 6)(x + 6)$$

Each side is $x + 6$.

Answer: $x + 6 \text{ cm}$

Marks: 1 mark for $(x + 6)(x + 6)$ or $(x + 6)^2$. 1 mark for $x + 6$.

- (b) Find an expression, in terms of x , for the perimeter of the square.
Give your answer without brackets.

A square has 4 equal sides.

$$\text{Perimeter} = 4(x + 6) = 4x + 24$$

Answer: $4x + 24 \text{ cm}$

11 (2 marks)

Factorise $x^2 - x - 30$

$-x$ means $-1x$.

Find two numbers that multiply to make -30 and add to make -1 .

The product is negative, so one number is positive and one is negative.

Factor pairs of 30: 1×30 , 2×15 , 3×10 , 5×6

$$-6 \times 5 = -30 \text{ and } -6 + 5 = -1 \checkmark$$

So the numbers are -6 and $+5$.

Answer: $(x - 6)(x + 5)$

Tip: $-x$ has no number in front, but it means $-1x$. The two numbers must add to -1 , so look for a factor pair of 30 that differs by 1.

Marks: 1 mark for two brackets $(x \pm a)(x \pm b)$ where the numbers multiply to -30 or add to -1 .
1 mark for $(x - 6)(x + 5)$.

12 (1 mark)

Priya says,

$'x^2 + 9$ factorises to $(x + 3)(x - 3)'$

Explain why Priya is wrong.

Expand Priya's brackets:

$$(x + 3)(x - 3) = x^2 - 3x + 3x - 9 = x^2 - 9$$

This is $x^2 - 9$, not $x^2 + 9$.

Only a *difference* of two squares (with a minus sign) factorises like this.

Answer: $(x + 3)(x - 3) = x^2 - 9$, not $x^2 + 9$

13 (2 marks)

The diagram shows a rectangle.



Diagram **NOT** accurately drawn

The area of the rectangle is $(x^2 + 11x + 24) \text{ cm}^2$

Use factorising to find the length of the rectangle. Give your answer in terms of x .

Area = length $\times$ width

Factorise the area: two numbers that multiply to 24 and add to 11 are 3 and 8.

$$\text{So } x^2 + 11x + 24 = (x + 3)(x + 8)$$

The width is $x + 3$, so the length is $x + 8$.

Answer: $x + 8$ cm

Marks: 1 mark for $(x + 3)(x + 8)$. 1 mark for $x + 8$.
