

Cubic and Reciprocal Graphs

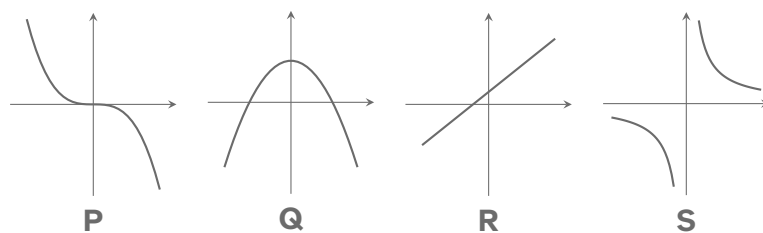
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Here are four graphs.



Match each equation to its graph.

$$y = -x^3 \quad \dots\dots\dots y = \frac{1}{x} \quad \dots\dots\dots y = 1 - x^2 \quad \dots\dots\dots$$

$y = -x^3$ is the cubic shape turned upside down: it goes from top-left to bottom-right. That is P.

$y = \frac{1}{x}$ has two separate parts that never touch the axes. That is S.

$y = 1 - x^2$ is a $\cap$ -shaped curve (the $-x^2$ turns it upside down). That is Q.

(R is a straight line.)

Answer: P, S, Q

Tip: $y = x^3$ goes **up** from left to right. $y = -x^3$ is the reflection, going **down** from left to right. These two are often mixed up.

Marks: 1 mark for each correct match.

2 (6 marks)

- (a) Complete the table of values for $y = x^3 - 4x$

x	-2	-1	0	1	2	3
y	0	3	0	-3	0	15

Substitute each value of x into $y = x^3 - 4x$.

$$x = -1: (-1)^3 - 4 \times (-1) = -1 + 4 = 3$$

$$x = 1: 1^3 - 4 \times 1 = 1 - 4 = -3$$

$$x = 3: 3^3 - 4 \times 3 = 27 - 12 = 15$$

Answer: 3, -3, 15

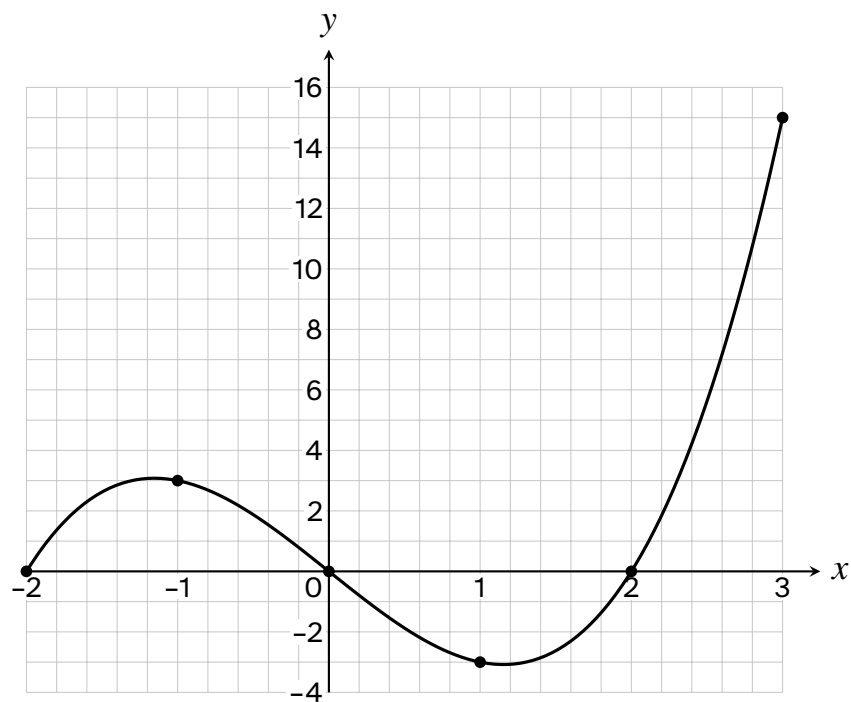
Tip: A cubic table is not symmetrical like a quadratic one, so don't copy values across.
 $(-1)^3 = -1$, and $-4 \times (-1) = +4$.

Marks: 2 marks for all three correct. 1 mark for one or two correct.

- (b) On the grid, draw the graph of $y = x^3 - 4x$ for values of x from -2 to 3

Plot the six points from the table.

Join them with one smooth curve.



Marks: 1 mark for at least 5 points plotted correctly. 1 mark for a smooth curve.

- (c) Write down the coordinates of the three points where the graph of $y = x^3 - 4x$ crosses the x -axis.

The graph crosses the x -axis where $y = 0$.

From the table, $y = 0$ when $x = -2$, $x = 0$ and $x = 2$.

On the x -axis the y -coordinate is 0.

Answer: $(-2, 0)$, $(0, 0)$, $(2, 0)$

Marks: 2 marks for all three points. 1 mark for two correct.

3 (5 marks)

- (a) Complete the table of values for $y = \frac{8}{x}$

x	1	2	4	5	8
y	8	4	2	1.6	1

$\frac{8}{x}$ means $8 \div x$.

$$x = 1: 8 \div 1 = 8$$

$$x = 4: 8 \div 4 = 2$$

$$x = 8: 8 \div 8 = 1$$

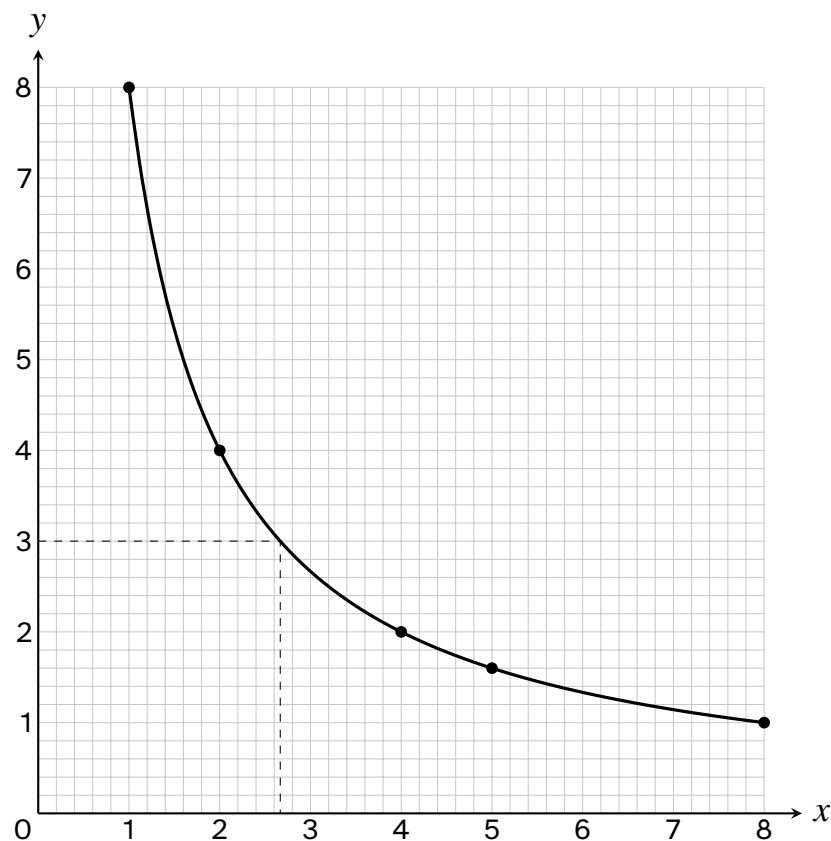
Answer: 8, 2, 1

Marks: 2 marks for all three correct. 1 mark for two correct.

- (b) On the grid, draw the graph of $y = \frac{8}{x}$ for values of x from 1 to 8

Plot the points: (1, 8), (2, 4), (4, 2), (5, 1.6), (8, 1).

Join them with one smooth curve.



Tip: Join the points with a smooth curve, not straight lines. The curve should get steeper as x gets closer to 1.

Marks: 1 mark for at least 4 points plotted correctly. 1 mark for a smooth curve.

- (c) Use your graph to find an estimate for the value of x when $y = 3$

Go across from 3 on the y -axis to the curve.

Then go down to the x -axis: $x \approx 2.7$

Check: $8 \div 2.7 = 2.96 \dots$, which is close to 3 ✓

Answer: 2.7 (accept 2.6 to 2.8)

4 (2 marks)

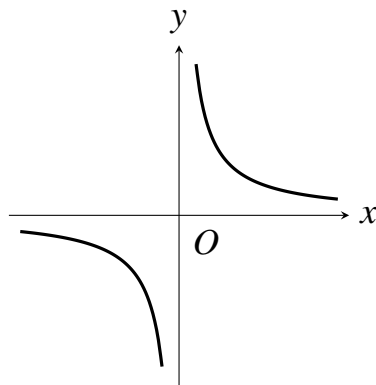
Sketch the graph of $y = \frac{1}{x}$

The graph has two separate curves.

When x is positive, y is positive, so one curve is in the top-right.

When x is negative, y is negative, so the other curve is in the bottom-left.

Neither curve touches the axes.



Tip: Draw **both** parts of the graph. Drawing only the top-right part, or a straight line, is a common mistake.

Marks: 1 mark for the correct shape in one of the two quarters. 1 mark for a fully correct sketch that does not touch the axes.

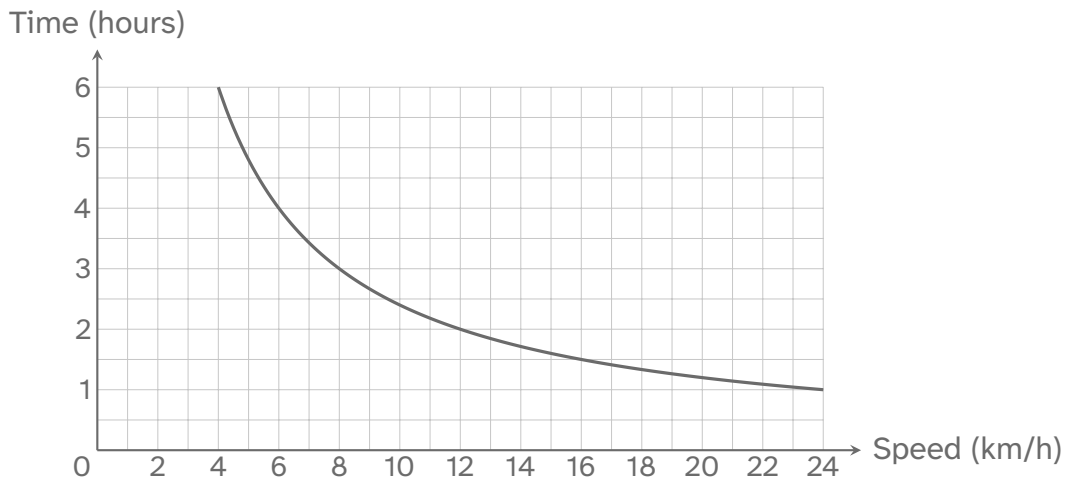
5 (5 marks)

Sam rides his bike for 24 km.

The time, t hours, that the ride takes is given by $t = \frac{24}{s}$

where s is Sam's average speed in km/h.

The graph of this formula is drawn on the grid.



- (a) Work out the value of t when $s = 16$

Substitute $s = 16$ into $t = \frac{24}{s}$:

$$t = \frac{24}{16} = 1.5$$

Answer: 1.5

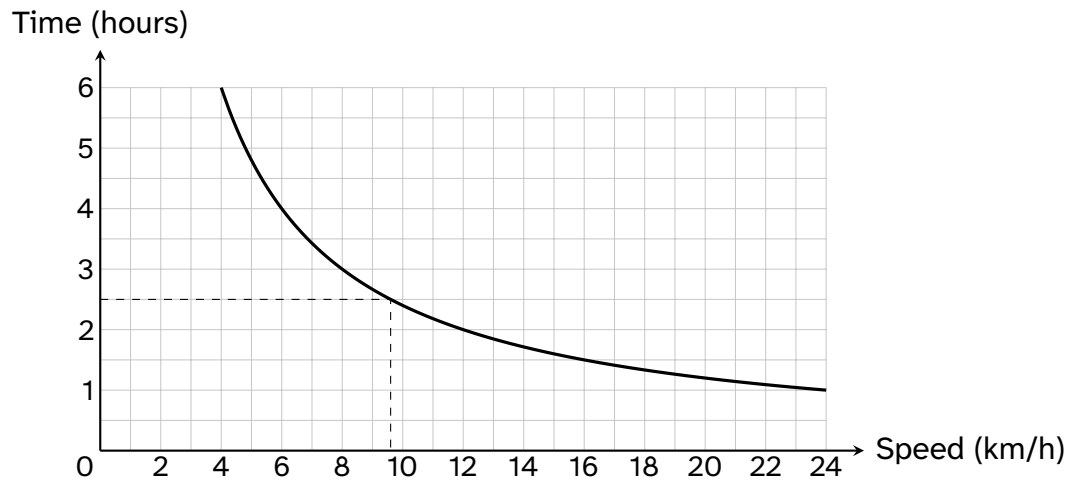
- (b) Use the graph to find an estimate for Sam's average speed when the ride takes 2 hours 30 minutes.

30 minutes is half an hour, so 2 hours 30 minutes = 2.5 hours.

Go across from 2.5 on the time axis to the curve (the dashed line).

Then go down to the speed axis: $s \approx 9.6$

Check with the formula: $24 \div 9.6 = 2.5$ ✓



Answer: 9.6 km/h (accept 9.4 to 9.8)

Tip: 2 hours 30 minutes is 2.5 hours, not 2.3 hours. There are 60 minutes in an hour, so 30 minutes is $\frac{30}{60} = 0.5$ of an hour.

Marks: 1 mark for using $t = 2.5$. 1 mark for an answer from 9.4 to 9.8.

- (c) Sam says, “If I ride twice as fast, the ride will take half the time.”
Is Sam correct? You must show how you get your answer.

Pick a speed and then double it, for example $s = 6$ and $s = 12$.

$$s = 6: t = 24 \div 6 = 4 \text{ hours}$$

$$s = 12: t = 24 \div 12 = 2 \text{ hours}$$

Doubling the speed from 6 to 12 halves the time from 4 hours to 2 hours.

This works for any speed, because $t = \frac{24}{s}$: doubling s halves $\frac{24}{s}$.

Answer: Yes, e.g. 6 km/h takes 4 hours and 12 km/h takes 2 hours

Marks: 1 mark for working out t for a speed and for double that speed. 1 mark for “Yes” with correct values.