

Congruence

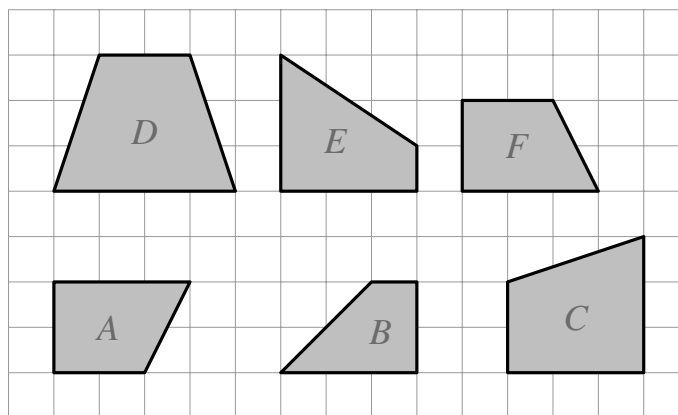
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Foundation tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Here are six shapes on a grid.



Two of the shapes are congruent.

Write down the letters of these two shapes.

Congruent shapes are exactly the same shape and size.

They can be turned round or flipped over.

A is a trapezium with parallel sides of 2 and 3 squares and a height of 2 squares.

F is a trapezium with parallel sides of 3 and 2 squares and a height of 2 squares.

F is *A* flipped upside down, so they are congruent.

Answer: *A* and *F*

2 (1 mark)

Here are four right-angled triangles.

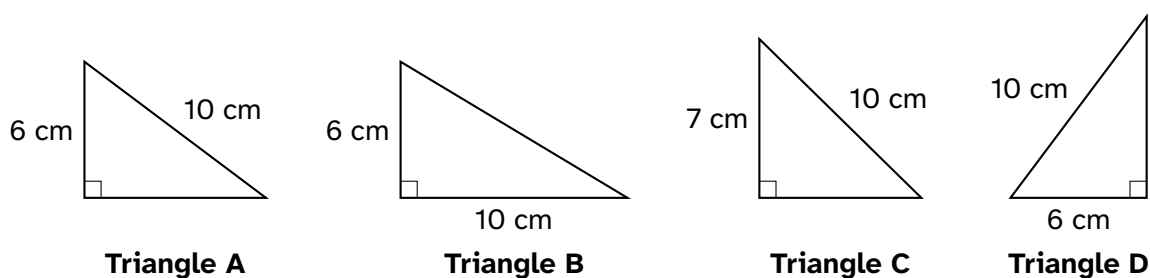


Diagram **NOT** accurately drawn

Which of triangles *B*, *C* and *D* is congruent to triangle *A*?

Triangle *A* has a right angle, a hypotenuse of 10 cm and another side of 6 cm.

Triangle *D* also has a right angle, a hypotenuse of 10 cm and another side of 6 cm.

So *A* and *D* are congruent (RHS).

In triangle *B* the 10 cm side is next to the right angle, so it is not the hypotenuse.

Triangle *C* has a 7 cm side, not 6 cm.

Answer: Triangle *D*

Tip: The hypotenuse is the side opposite the right angle. Check which side is 10 cm, not just which numbers appear.

3 (2 marks)

Write down the letters of the **two** triangles that are congruent.

<i>P</i>	angles 40° and 70° , with a side of 6 cm between them
<i>Q</i>	sides 6 cm and 7 cm, with an angle of 40° between them
<i>R</i>	angles 70° and 40° , with a side of 6 cm between them
<i>S</i>	angles 40° , 70° and 70°

P and *R* both have angles of 40° and 70° with a 6 cm side between them.

Two angles and the side between them is the condition ASA.

Q has two sides and one angle, which does not match P or R .

S has only angles and no lengths, so it could be any size.

Answer: P and R

Marks: 2 marks for both correct. 1 mark for one correct.

4 (3 marks)

Write down the condition (SSS, SAS, ASA or RHS) that shows each pair of triangles is congruent.

- (a) All three pairs of sides are equal.

Side, Side, Side.

Answer: SSS

- (b) Both triangles are right-angled. The hypotenuses are equal and one other pair of sides is equal.

Right angle, Hypotenuse, Side.

Answer: RHS

- (c) Two pairs of sides are equal and the angles between them are equal.

Side, Angle, Side (the angle is between the two sides).

Answer: SAS

5 (1 mark)

Two triangles each have angles of 50° , 60° and 70°

Rana says the triangles must be congruent.

Is Rana correct?

Give a reason for your answer.

No.

Having the same angles means the triangles are the same shape.

But one could be bigger than the other, so they could be different sizes.

Answer: No, they could be different sizes

6 (2 marks)

Triangle ABC is congruent to triangle PQR , with A matching P , B matching Q and C matching R .

$AB = 6$ cm, $BC = 8$ cm, angle $ABC = 70^\circ$ and angle $BAC = 60^\circ$

(a) Write down the length of QR .

Q matches B and R matches C .

So QR matches BC , which is 8 cm.

Answer: 8 cm

(b) Work out the size of angle PRQ .

Angle PRQ matches angle ACB .

Angles in a triangle add up to 180° .

Angle $ACB = 180 - 70 - 60 = 50^\circ$

Answer: 50°

7 (2 marks)

B , C and D lie on a straight line.

Angle $ABC = \text{angle } CDE = 90^\circ$

Triangle ABC is congruent to triangle CDE .

$AB = 4$ cm and $BD = 11$ cm

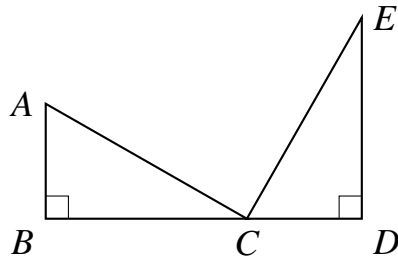


Diagram **NOT** accurately drawn

Work out the length of DE .

In the congruent triangles, A matches C , B matches D and C matches E .

So $CD = AB = 4$ cm

$BC = BD - CD = 11 - 4 = 7$ cm

DE matches BC , so $DE = 7$ cm

Answer: 7 cm

Tip: Match the letters in order: ABC matches CDE , so AB matches CD and BC matches DE .

Marks: 1 mark for $CD = 4$ or $BC = 7$. 1 mark for 7.

8 (2 marks)

Triangle 1 has angles of 50° and 60° , with a side of 8 cm between them.

Triangle 2 has angles of 60° and 70° , with a side of 8 cm between them.

Mia says the triangles are congruent because of ASA.

Is Mia correct?

You must give a reason for your answer.

Angles in a triangle add up to 180° .

Triangle 1: third angle = $180 - 50 - 60 = 70^\circ$

Its 8 cm side is between 50° and 60° , so it is opposite the 70° angle.

Triangle 2: third angle = $180 - 60 - 70 = 50^\circ$

Its 8 cm side is between 60° and 70° , so it is opposite the 50° angle.

The 8 cm sides are not in matching positions, so the triangles are not congruent.

Answer: No: the 8 cm side is opposite 70° in one triangle and opposite 50° in the other

Tip: For ASA, the equal side must be between the **same** two angles in both triangles.

Marks: 1 mark for finding the third angle of either triangle. 1 mark for No with a correct reason.

9 (3 marks)

The quadrilateral $ABCD$ is made from two congruent triangles.

Triangle ABD is congruent to triangle CDB (A matches C , B matches D , D matches B).

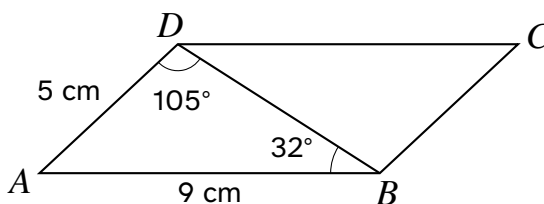


Diagram **NOT** accurately drawn

- (a) Write down the length of BC .

Match the letters in order: A matches C , B matches D and D matches B .

So side CB in triangle CDB matches side AD in triangle ABD .

$$BC = AD = 5 \text{ cm}$$

Answer: 5 cm

Tip: Use the order of the letters, not the look of the diagram. CDB matches ABD , so CB matches AD .

- (b) Work out the size of angle ABC .

Angle ABC is made of angle ABD and angle DBC .

Angle DBC is the angle at B in triangle CDB .

B matches D , so angle $DBC = \text{angle } ADB = 105^\circ$

Angle $ABC = 32^\circ + 105^\circ = 137^\circ$

Answer: 137°

Tip: Angle ABC is the whole angle at B , not just the 32° part. Find the other part using the congruent triangle.

Marks: 1 mark for angle $DBC = 105^\circ$. 1 mark for 137.

10 (3 marks)

The large triangle ABC is made from four congruent triangles.

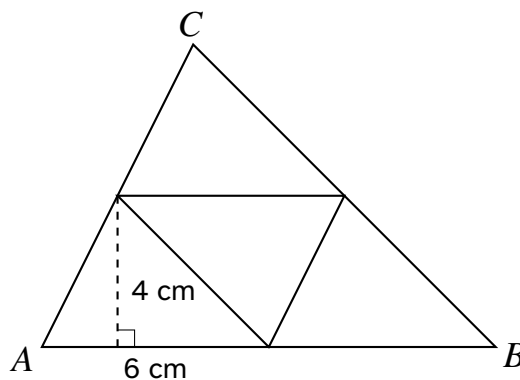


Diagram **NOT** accurately drawn

(a) Work out the area of triangle ABC .

$$\text{Area of one small triangle} = \frac{1}{2} \times 6 \times 4 = 12\text{ cm}^2$$

The four small triangles are congruent, so they all have the same area.

$$\text{Area of triangle } ABC = 4 \times 12 = 48\text{ cm}^2$$

Answer: 48 cm^2

Tip: There are **four** small triangles, so multiply by 4. Don't forget the $\frac{1}{2}$ when you find the area of one triangle.

Marks: 1 mark for 12. 1 mark for 48.

- (b) The perimeter of triangle ABC is 32 cm.
Work out the perimeter of one of the small triangles.

Each side of triangle ABC is made from two equal sides of the small triangles.

So each side of a small triangle is half of a side of ABC .

Perimeter of a small triangle = $32 \div 2 = 16$ cm

Answer: 16 cm

Tip: Dividing by 4 is wrong. Each small triangle has sides half as long as the big triangle, so its perimeter is half of 32.
