

Volume and Surface Area of Prisms and Cylinders

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

A cylinder has diameter 6 cm and height 10 cm.

Circle the area, in cm^2 , of its curved surface.

Options: 30π , 60π , 90π , 120π

The curved surface unrolls into a rectangle.

Its length is the circumference, πd , and its width is the height.

Curved surface area = $\pi \times 6 \times 10 = 60\pi$

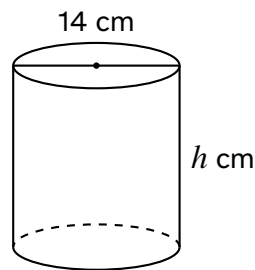
Answer: 60π

Tip: 90π is the volume ($\pi \times 3^2 \times 10$), not the curved surface area.

2 (5 marks)

A solid cylinder has diameter 14 cm.

The volume of the cylinder is $1470\pi \text{ cm}^3$.



Not drawn accurately

- (a) Work out the height of the cylinder.

The radius is $14 \div 2 = 7 \text{ cm}$.

$$\text{Volume} = \pi r^2 h$$

$$\pi \times 7^2 \times h = 1470\pi$$

$$49\pi h = 1470\pi$$

$$h = 1470 \div 49 = 30$$

Answer: 30 cm

Tip: Halve the diameter first. Using 14^2 gives a height of 7.5 cm, which is wrong.

Marks: 1 mark for $\pi \times 7^2 \times h = 1470\pi$. 1 mark for 30.

- (b) Work out the total surface area of the cylinder.
Give your answer to 3 significant figures.

A cylinder has two circular ends and one curved surface.

$$\text{Two ends: } 2 \times \pi \times 7^2 = 98\pi$$

$$\text{Curved surface} = 2\pi rh = 2 \times \pi \times 7 \times 30 = 420\pi$$

$$\text{Total} = 98\pi + 420\pi = 518\pi = 1627.3 \dots$$

$$= 1630 \text{ (3 s.f.)}$$

Answer: 1630 cm^2

Tip: Include **both** circular ends. Use your height from part (a) in the curved surface, $2\pi rh$.

Marks: 1 mark for the two ends (98π). 1 mark for the curved surface (420π). 1 mark for 1630.

3 (2 marks)

A cylinder has volume $72\pi \text{ cm}^3$ and height 8 cm.
Work out its radius.

$$\text{Volume} = \pi r^2 h$$

$$\pi \times r^2 \times 8 = 72\pi$$

Divide both sides by 8π : $r^2 = 9$

$$r = \sqrt{9} = 3$$

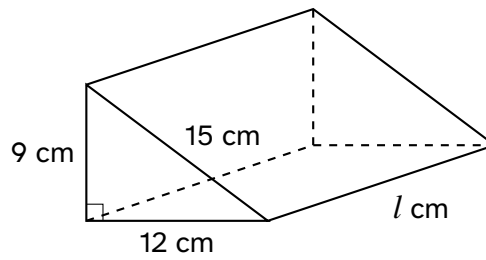
Answer: 3 cm

Tip: The cylinder formula is $\pi r^2 h$, with no $\frac{1}{3}$ (that is for a cone). Remember to square root at the end.

Marks: 1 mark for $\pi r^2 \times 8 = 72\pi$. 1 mark for 3.

4 (3 marks)

The cross section of a prism is a right-angled triangle with sides 9 cm, 12 cm and 15 cm.



Not drawn accurately

The total surface area of the prism is 828 cm^2 .

Work out the length of the prism.

The prism has 2 triangles and 3 rectangles.

$$\text{Each triangle} = \frac{1}{2} \times 9 \times 12 = 54$$

$$\text{Two triangles} = 2 \times 54 = 108 \text{ cm}^2$$

$$\text{The three rectangles have total area } 828 - 108 = 720 \text{ cm}^2$$

Let the length be l . The rectangles are $9 \times l$, $12 \times l$ and $15 \times l$.

$$(9 + 12 + 15) \times l = 36l = 720$$

$$l = 720 \div 36 = 20$$

Answer: 20 cm

Tip: The three rectangles all have width l , so their total area is (perimeter of the triangle) $\times l$. Don't forget there are **two** triangles.

Marks: 1 mark for the two triangles (108). 1 mark for $36l = 828 - 108$. 1 mark for 20.

5 (4 marks)

A cuboid tank measures 40 cm by 30 cm by 20 cm.

The tank is full of water.

All the water is poured into an empty cylinder of radius 15 cm.

Work out the depth of water in the cylinder.

$$\text{Volume of water} = 40 \times 30 \times 20 = 24\,000 \text{ cm}^3$$

The water in the cylinder has the same volume.

Let the depth be h .

$$\pi \times 15^2 \times h = 24\,000$$

$$\text{Area of the circular base} = \pi \times 15^2 = 706.8 \dots \text{ cm}^2$$

$$h = 24\,000 \div 706.8 \dots = 33.95 \dots$$

$$= 34.0 \text{ (3 s.f.)}$$

Answer: 34.0 cm

Marks: 1 mark for $40 \times 30 \times 20$. 1 mark for $\pi \times 15^2$. 1 mark for $24\,000 \div$ your area. 1 mark for 34.0 (or 33.95 ...).

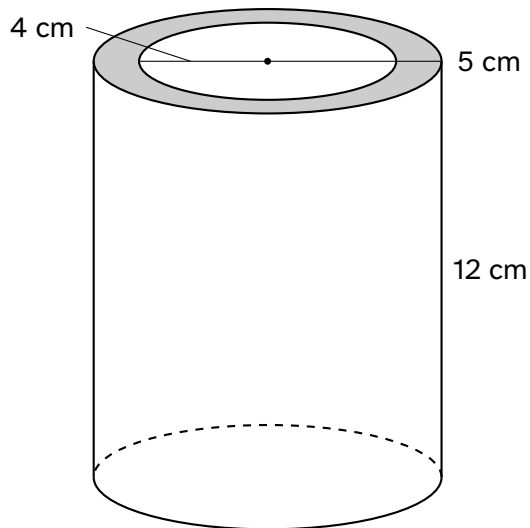
6 (4 marks)

A metal pipe is a hollow cylinder.

The outer radius is 5 cm.

The inner radius is 4 cm.

The pipe is 12 cm long.



Not drawn accurately

Work out the volume of metal in the pipe.

Give your answer to 3 significant figures.

The cross section of the pipe is a ring: the outer circle minus the inner circle.

$$\text{Outer circle} = \pi \times 5^2 = 25\pi$$

$$\text{Inner circle} = \pi \times 4^2 = 16\pi$$

$$\text{Area of cross section} = 25\pi - 16\pi = 9\pi \text{ cm}^2$$

$$\text{Volume} = \text{area of cross section} \times \text{length} = 9\pi \times 12 = 108\pi$$

$$= 339.29 \dots$$

$$= 339 \text{ (3 s.f.)}$$

Answer: 339 cm³

Tip: Subtract the areas, not the radii. $\pi \times (5 - 4)^2 \times 12$ is a common wrong method.

Marks: 1 mark for $\pi \times 5^2$ or $\pi \times 4^2$. 1 mark for 9π (the area of the ring). 1 mark for $\times 12$. 1 mark for 339.

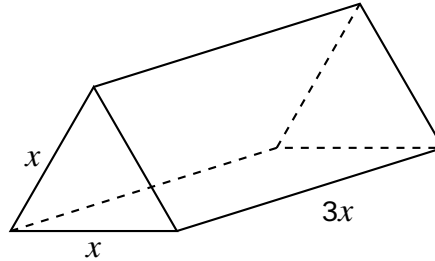
7 (4 marks)

A candle is a prism.

All lengths are in centimetres.

The cross section is an equilateral triangle with side x

The length of the candle is $3x$



Not drawn accurately

The volume of the candle must be at least 500 cm^3

Work out the smallest possible integer value of x .

You must show your working.

Area of an equilateral triangle with side x : $\frac{1}{2} \times x \times x \times \sin 60^\circ = \frac{\sqrt{3}}{4}x^2$

Volume = area of cross section $\times$ length

$$= \frac{\sqrt{3}}{4}x^2 \times 3x = \frac{3\sqrt{3}}{4}x^3 = 1.299 \dots x^3$$

$$1.299 \dots x^3 \geq 500$$

$$x^3 \geq 384.9 \dots$$

$$x \geq 7.27 \dots$$

The smallest whole number that is at least $7.27 \dots$ is 8.

Answer: 8

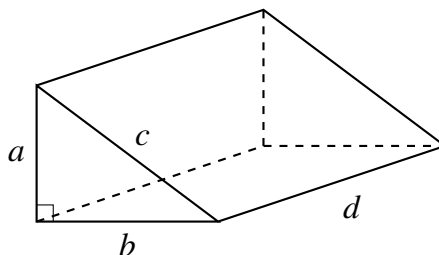
Tip: Here you round **up**: $x = 7$ gives a volume of only 445.6 cm^3 , which is less than 500.

Marks: 1 mark for the area of the triangle, $\frac{\sqrt{3}}{4}x^2$ (or $\frac{1}{2}x^2 \sin 60^\circ$). 1 mark for the volume $\frac{3\sqrt{3}}{4}x^3$. 1 mark for $x^3 \geq 384.9 \dots$ or $x \geq 7.27 \dots$ 1 mark for 8.

8 (5 marks)

Here is a right-angled triangular prism.

The ratio of the edges is $a:b:c:d = 5:12:13:10$



Not drawn accurately

The volume of the prism is 2400 cm^3

Work out the total length of all of the edges of the prism.

Use the ratio to write each length in terms of k :

$$a = 5k, b = 12k, c = 13k, d = 10k$$

Volume = area of triangle $\times$ length

$$= \frac{1}{2} \times 5k \times 12k \times 10k = 300k^3$$

$$300k^3 = 2400$$

$$k^3 = 8$$

$$k = \sqrt[3]{8} = 2$$

So $a = 10$, $b = 24$, $c = 26$ and $d = 20$.

The prism has 2 triangles (3 edges each) and 3 edges of length d .

$$\text{Total} = 2 \times (10 + 24 + 26) + 3 \times 20$$

$$= 120 + 60 = 180$$

Answer: 180 cm

Tip: Do not divide 2400 by $5 + 12 + 13 + 10$. Volume is length $\times$ length $\times$ length, so you get k^3 and must cube root. There are three edges of length d , not two.

Marks: 1 mark for the lengths in terms of k . 1 mark for $300k^3 = 2400$. 1 mark for $k = 2$. 1 mark for $2(a + b + c) + 3d$. 1 mark for 180.