

Upper and Lower Bounds

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

The mass of a parcel is 4.7 kg to 1 decimal place.

Circle the error interval for the mass, m kg.

Options: $4.6 \leq m < 4.8$, $4.65 \leq m < 4.75$, $4.65 \leq m \leq 4.75$, $4.7 \leq m < 4.8$

1 decimal place means to the nearest 0.1

Half of 0.1 is 0.05

Lower bound: $4.7 - 0.05 = 4.65$

Upper bound: $4.7 + 0.05 = 4.75$

4.75 itself would round up to 4.8, so use $<$ at the top.

Answer: $4.65 \leq m < 4.75$

2 (3 marks)

A floor tile is 1.2 cm thick, correct to 1 decimal place.

(a) Complete the error interval for the thickness, t cm, of one tile.

To 1 decimal place means to the nearest 0.1

Half of 0.1 is 0.05

Lower bound: $1.2 - 0.05 = 1.15$

Upper bound: $1.2 + 0.05 = 1.25$

Answer: $1.15 \leq t < 1.25$

Marks: 1 mark for 1.15. 1 mark for 1.25.

- (b) 15 of these tiles are stacked on top of each other.

Could the height of the stack be 19 cm?

Tick a box.

Give a reason for your answer.

Use the bounds from part (a) and multiply by 15.

Smallest possible height: $15 \times 1.15 = 17.25$ cm

Largest possible height: $15 \times 1.25 = 18.75$ cm

19 cm is more than 18.75 cm, so the stack cannot be 19 cm tall.

Answer: No, the height must be less than 18.75 cm

Tip: Multiply the bounds, not the rounded value. $15 \times 1.2 = 18$ does not tell you the largest possible height. Use the upper bound, 1.25 cm.

3 (4 marks)

- (a) $x = 3.7$ truncated to 1 decimal place.

Complete the error interval for x .

Truncating means chopping off the extra digits (no rounding).

Any number from 3.7 up to 3.7999...truncates to 3.7

So x is at least 3.7 and less than 3.8

Answer: $3.7 \leq x < 3.8$

Marks: 1 mark for 3.7. 1 mark for 3.8.

- (b) When w is truncated to a whole number, the answer is 12

Sacha writes the error interval due to truncation as $11.5 \leq w < 12.5$

Give a reason why Sacha is wrong and state the correct error interval for w .

Sacha has used the error interval for *rounding*, not truncating.

A number below 12 cannot truncate to 12. For example, 11.8 truncates to 11.

Any number from 12 up to 12.999...truncates to 12.

Correct interval: $12 \leq w < 13$

Answer: Sacha has rounded, not truncated; $12 \leq w < 13$

Marks: 1 mark for a correct reason, e.g. 11.5 truncates to 11. 1 mark for $12 \leq w < 13$.

4 (2 marks)

A rectangle has length 12 cm and width 7 cm, both correct to the nearest centimetre. Work out the upper bound for the area of the rectangle.

To the nearest cm, so add half of 1 cm (0.5 cm) for the upper bounds.

Upper bound of length: 12.5 cm

Upper bound of width: 7.5 cm

Largest area: $12.5 \times 7.5 = 93.75 \text{ cm}^2$

Answer: 93.75 cm^2

Marks: 1 mark for using 12.5 and 7.5. 1 mark for 93.75.

5 (3 marks)

A runner runs 100 m, correct to the nearest metre. Her time is 12.4 seconds, correct to 1 decimal place. Work out the lower bound for her average speed. Give your answer to 3 decimal places.

Speed = distance $\div$ time

Bounds of distance: 99.5 m and 100.5 m

Bounds of time: 12.35 s and 12.45 s

For the **smallest** speed: smallest distance $\div$ **largest** time

$$\frac{99.5}{12.45} = 7.9919 \dots$$

To 3 decimal places: 7.992 m/s

Answer: 7.992 m/s

Tip: When you divide, the lower bound uses the **upper** bound of the bottom number. Dividing by a bigger number gives a smaller answer.

Marks: 1 mark for 99.5 and 12.45. 1 mark for $99.5 \div 12.45$. 1 mark for 7.992.

6 (3 marks)

$$r = \frac{a^2}{c + 2}$$

$a = 6.4$ correct to 1 decimal place.

$c = 3.8$ correct to 2 significant figures.

Work out the upper bound for r .

Give your answer to 3 significant figures.

Bounds of a : $6.35 \leq a < 6.45$

Bounds of c : $3.75 \leq c < 3.85$

To make r as large as possible:

make the top as big as possible, so use the largest a (6.45),

and make the bottom as small as possible, so use the **smallest** c (3.75).

$$r = \frac{6.45^2}{3.75 + 2} = \frac{41.6025}{5.75} = 7.2352 \dots$$

To 3 significant figures: 7.24

Answer: 7.24

Tip: c is on the bottom of the fraction. A smaller bottom gives a bigger answer, so the upper bound of r uses the **lower** bound of c .

Marks: 1 mark for 6.45 and 3.75. 1 mark for $\frac{6.45^2}{3.75+2}$. 1 mark for 7.24.

7 (3 marks)

$m = 0.6$ to 1 significant figure

$n = 40$ to the nearest 5

Work out the lower bound for $n - 30m$

You must show your working.

$m = 0.6$ to 1 significant figure, so it is rounded to the nearest 0.1

Half of 0.1 is 0.05, so $0.55 \leq m < 0.65$

$n = 40$ to the nearest 5, so $37.5 \leq n < 42.5$

For the smallest answer: use the smallest n and take away as much as possible, so use the **largest** m .

$$37.5 - 30 \times 0.65 = 37.5 - 19.5 = 18$$

Answer: 18

Tip: When you subtract, the lower bound uses the **upper** bound of the part being taken away.

Marks: 1 mark for 0.65 or 37.5. 1 mark for $37.5 - 30 \times 0.65$. 1 mark for 18.

8 (5 marks)

A fish tank is a cuboid.

Its inside measurements are 80 cm by 50 cm by 40 cm, each to the nearest centimetre.

The empty tank is filled with water from a hose.

Water flows from the hose at 12 litres per minute, to the nearest litre per minute.

Work out the upper bound for the time it takes to fill the tank.

Give your answer in minutes to 3 significant figures.

You must show your working.

Each length is to the nearest cm, so the upper bounds are 80.5 cm, 50.5 cm and 40.5 cm.

$$\text{Upper bound of the volume: } 80.5 \times 50.5 \times 40.5 = 164\,642.625 \text{ cm}^3$$

$$1000 \text{ cm}^3 = 1 \text{ litre, so this is } 164.642\,625 \text{ litres.}$$

The rate is to the nearest litre per minute, so its lower bound is 11.5 litres per minute.

For the longest time: largest volume $\div$ **smallest** rate

$$\frac{164.642\,625}{11.5} = 14.316 \dots$$

To 3 significant figures: 14.3 minutes

Answer: 14.3 minutes

Tip: The longest time comes from the biggest volume and the **slowest** rate. Using the upper bound of the rate (12.5) gives the shortest time instead.

Marks: 1 mark for 80.5, 50.5 and 40.5. 1 mark for $80.5 \times 50.5 \times 40.5$. 1 mark for dividing by 1000 to change to litres. 1 mark for 11.5. 1 mark for 14.3.

9 (4 marks)

Kofi has a small metal bar.

- The mass of the bar is 395 g, to the nearest 5 g.
- The volume of the bar is 20 cm^3 , to the nearest cm^3 .

Pure gold has a density of 19.3 g/cm^3 .

Kofi says, " $395 \div 20 = 19.75$, so my bar cannot be pure gold."

Is Kofi correct?

Tick a box.

Show working to support your answer.

Density = mass $\div$ volume

Mass is to the nearest 5 g, so its bounds are 392.5 g and 397.5 g.

Volume is to the nearest cm^3 , so its bounds are 19.5 cm^3 and 20.5 cm^3 .

Kofi's value, 19.75, is above 19.3, so check the **smallest** possible density.

Smallest density: smallest mass $\div$ **largest** volume

$$\frac{392.5}{20.5} = 19.146 \dots \text{ g/cm}^3$$

19.146... is less than 19.3, and 19.75 is more than 19.3

So the density could be exactly 19.3 g/cm^3 : the bar could be pure gold.

Kofi is not correct.

Answer: No. The lower bound of the density is 19.1 g/cm^3 (3 s.f.), which is less than 19.3, so the bar could be pure gold.

Tip: Dividing the rounded values (or $392.5 \div 19.5$) gives a density above 19.3, which wrongly suggests the bar cannot be gold. For the smallest density, divide by the **largest** volume.

Marks: 1 mark for 392.5 or 20.5. 1 mark for $392.5 \div 20.5$. 1 mark for 19.1 or better (19.146...). 1 mark for No, with 19.146... compared with 19.3.
