

Trigonometric Graphs

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the value of $\sin 270^\circ$

Options: -1 , 0 , 0.5 , 1

On the graph of $y = \sin x$, the curve goes down from $(180, 0)$ to its lowest point at $x = 270$.

The lowest value of $\sin x$ is -1 , so $\sin 270^\circ = -1$.

Answer: -1

2 (2 marks)

Here are four statements about the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0 \leq x \leq 360$

For each statement, tick the correct box.

	True	False
$y = \cos x$ has a minimum point at $(180, -1)$	✓	
$y = \tan x$ has an asymptote at $x = 180$		✓
$y = \sin x$ and $y = \cos x$ cross the x -axis at the same points		✓
The largest value of $\sin x$ is 1	✓	

Minimum of $y = \cos x$: $\cos 180^\circ = -1$, the lowest value of $\cos$. **True**

Asymptote at $x = 180$: $\tan 180^\circ = 0$, so the graph passes through $(180, 0)$. The asymptotes are at $x = 90$ and $x = 270$. **False**

Crossing the x -axis: $\sin x = 0$ at $0, 180, 360$, but $\cos x = 0$ at $90, 270$. **False**

Largest value of $\sin x$: the maximum is $\sin 90^\circ = 1$. **True**

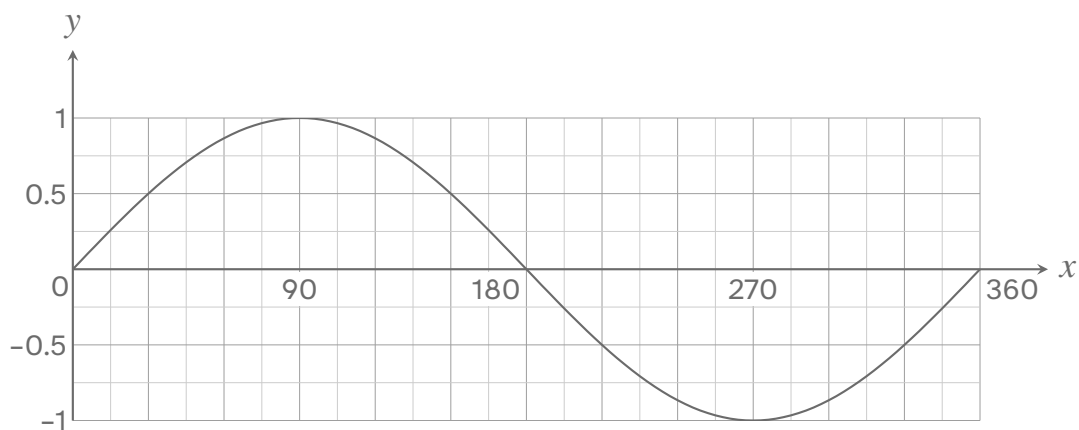
Answer: True, False, False, True

Tip: Learn the key points: $\sin$ and $\cos$ take the values $0, 1$ and -1 at $0^\circ, 90^\circ, 180^\circ, 270^\circ$ and 360° , and $\tan$ has asymptotes at 90° and 270° . A quick sketch helps.

Marks: 2 marks for all four correct. 1 mark for two or three correct.

3 (4 marks)

Here is the graph of $y = \sin x$ for $0 \leq x \leq 360$



- (a) Write down the coordinates of the maximum point.

The highest point of the curve is at $x = 90$, where $y = \sin 90^\circ = 1$.

Answer: (90, 1)

- (b) Use the graph to solve $\sin x = 0.5$ for $0 \leq x \leq 360$

Draw the line $y = 0.5$ across the graph.

It crosses the curve twice, so there are two solutions.

First solution: $x = 30$ ($\sin 30^\circ = 0.5$)

The sine curve is symmetrical about $x = 90$, so the other solution is $180 - 30 = 150$.

Answer: $x = 30^\circ$ and 150°

Tip: There are usually **two** solutions between 0° and 360° . Draw the horizontal line and count how many times it crosses the curve.

Marks: 1 mark for each correct solution.

- (c) Write down the number of solutions of $\sin x = -0.8$ for $0 \leq x \leq 360$

Imagine the line $y = -0.8$ across the graph.

It crosses the curve twice (once between 180 and 270 , once between 270 and 360).

Answer: 2

4 (1 mark)

Circle the value of x , between 0° and 360° , for which $\cos x = \cos 70^\circ$ and $x \neq 70^\circ$

Options: 110° , 250° , 290° , 340°

The cosine graph is symmetrical about the line $x = 180$.

So $\cos 70^\circ = \cos(360^\circ - 70^\circ) = \cos 290^\circ$

Answer: 290°

5 (1 mark)

$\sin 40^\circ = 0.643$ to 3 decimal places.

Write down another angle between 0° and 360° with $\sin x = 0.643$

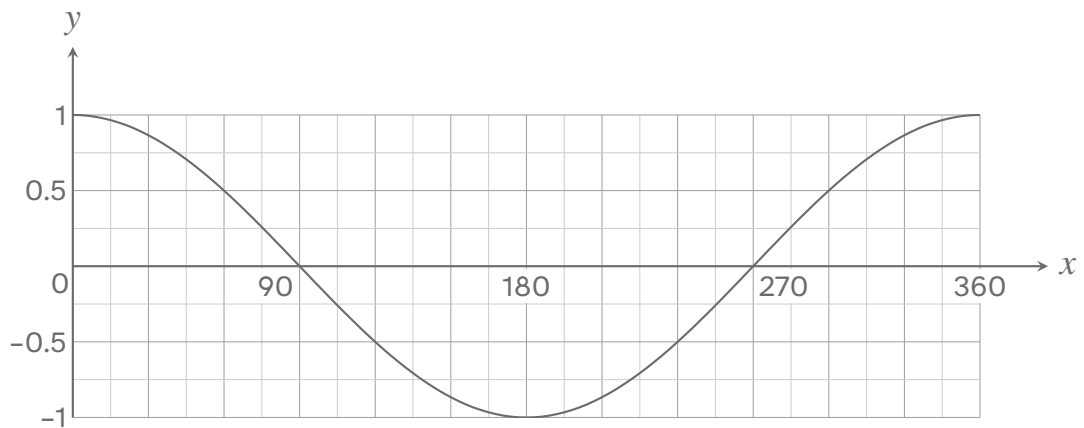
The sine graph is symmetrical about the line $x = 90$.

So $\sin 40^\circ = \sin(180^\circ - 40^\circ) = \sin 140^\circ$

Answer: 140

6 (4 marks)

Here is the graph of $y = \cos x$ for $0 \leq x \leq 360$



- (a) Solve $\cos x = -0.5$ for $0 \leq x \leq 360$

Use the calculator: $\cos^{-1}(-0.5) = 120^\circ$

The cosine graph is symmetrical about $x = 180$.

So the other solution is $360 - 120 = 240^\circ$

Answer: 120° and 240°

Marks: 1 mark for each correct solution.

- (b) $\cos 35^\circ = 0.819$ to 3 decimal places.

Write down the value of $\cos 325^\circ$

The cosine graph is symmetrical about $x = 180$.

$325 = 360 - 35$, so 325° is the same distance from 360° as 35° is from 0° .

So $\cos 325^\circ = \cos 35^\circ = 0.819$

Answer: 0.819

- (c) Write down the value of $\cos 215^\circ$

$$215 = 180 + 35$$

Between 180° and 270° the cosine graph is below the x -axis, so $\cos 215^\circ$ is negative.

The graph between 180° and 270° is the graph between 0° and 90° turned upside down, so $\cos 215^\circ = -\cos 35^\circ$.

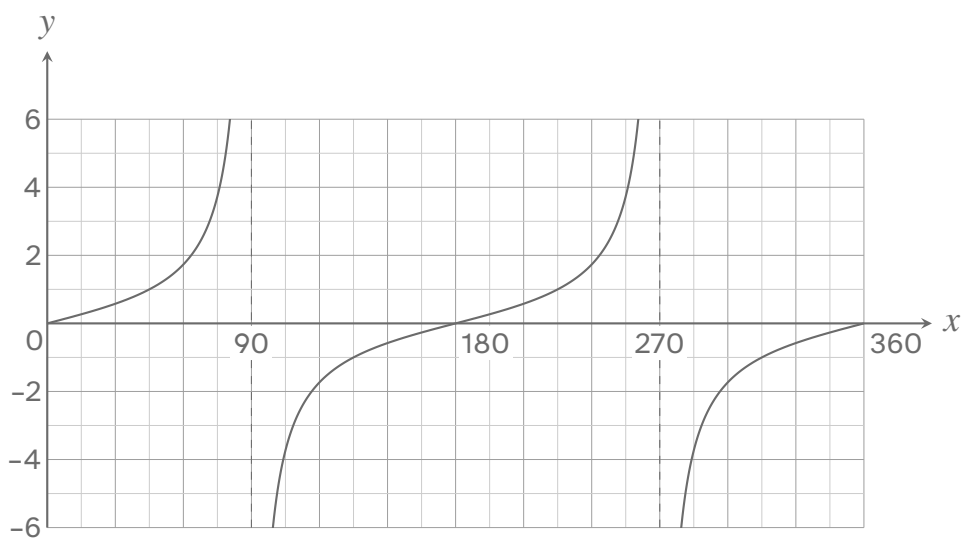
$$\cos 215^\circ = -0.819$$

Answer: -0.819

Tip: Check the sign on the graph first: $\cos$ is negative for angles between 90° and 270° . Then use the symmetry to get the size, 0.819.

7 (5 marks)

Here is the graph of $y = \tan x$ for $0 \leq x \leq 360$



- (a) Write down the equations of the asymptotes of the graph.

An asymptote is a line the graph gets closer and closer to but never meets.

$\tan 90^\circ$ and $\tan 270^\circ$ are undefined.

So the asymptotes are the vertical lines at $x = 90$ and $x = 270$.

Answer: $x = 90$ and $x = 270$

Marks: 1 mark for each equation.

- (b) You are given that $\tan x = 3$ when $x = 71.57^\circ$ (to 2 decimal places).
Work out the other value of x where $\tan x = 3$ and $0^\circ \leq x \leq 360^\circ$

The tan graph repeats every 180° .

So $\tan(71.57^\circ + 180^\circ) = 3$ as well.

$$71.57 + 180 = 251.57$$

Answer: 251.57°

Tip: Tan repeats every 180° (not 360° like sin and cos). Add 180° to find the next solution.

- (c) Work out the two values of x where $\tan x = -3$ and $0^\circ \leq x \leq 360^\circ$

By symmetry of the tan graph, $\tan(180^\circ - 71.57^\circ) = -3$.

$$180 - 71.57 = 108.43$$

Add 180 for the next solution: $108.43 + 180 = 288.43$

Answer: 108.43° and 288.43°

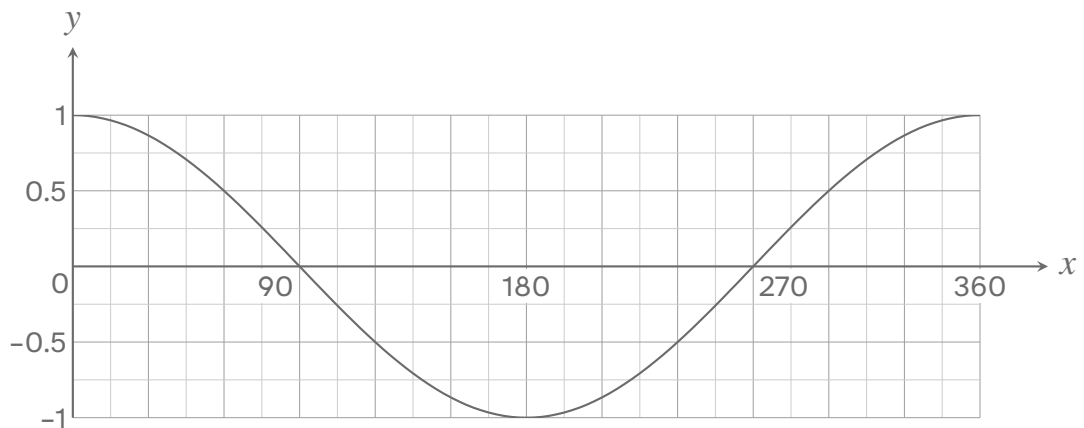
Marks: 1 mark for each correct value.

8 (2 marks)

θ is an acute angle.

$$\cos \theta = c$$

Here is the graph of $y = \cos x$ for $0 \leq x \leq 360$



For each expression, tick whether it is equal to c or equal to $-c$

	Equal to c	Equal to $-c$
$\cos(180^\circ - \theta)$		✓
$\cos(360^\circ - \theta)$	✓	
$\cos(180^\circ + \theta)$		✓
$\cos(-\theta)$	✓	

Use the symmetry of the cosine graph.

$\cos(180^\circ - \theta)$: the graph has rotational symmetry about $(90, 0)$, so this is $-\cos \theta = -c$

$\cos(360^\circ - \theta)$: the graph is symmetrical about $x = 180$, so this is $\cos \theta = c$

$\cos(180^\circ + \theta)$: this is the same distance from 180° as $180^\circ - \theta$, so it equals $\cos(180^\circ - \theta) = -c$

$\cos(-\theta)$: the graph is symmetrical about the y -axis, so this is $\cos \theta = c$

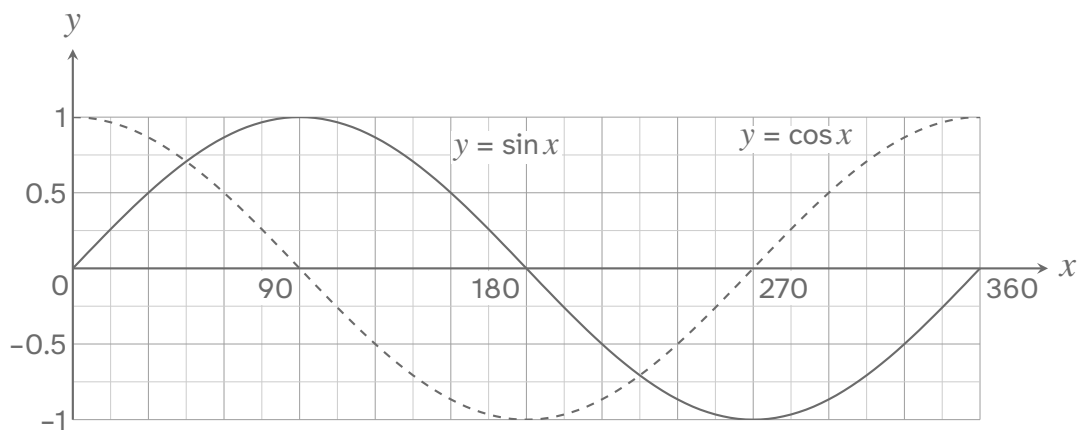
Answer: $-c, c, -c, c$

Tip: Try it with a real angle on your calculator, e.g. $\theta = 30^\circ$: $\cos 30^\circ = 0.866$, and check which expressions give 0.866 and which give -0.866 .

Marks: 2 marks for all four correct. 1 mark for two or three correct.

9 (2 marks)

The graphs of $y = \sin x$ and $y = \cos x$ are drawn on the grid for $0 \leq x \leq 360$



- (a) Write down the number of points where the two graphs intersect.

Look for the places where the two curves cross each other.

They cross once between 0° and 90° and once between 180° and 270° .

Answer: 2

- (b) Write down the x -coordinates of the points where the two graphs intersect.

The graphs cross where $\sin x = \cos x$.

From the graph, the first crossing is halfway between 0 and 90, at $x = 45$ ($\sin 45^\circ = \cos 45^\circ = 0.707 \dots$).

The second crossing is 180° later, at $x = 225$ ($\sin 225^\circ = \cos 225^\circ = -0.707 \dots$).

Answer: $x = 45$ and $x = 225$

Tip: Read the x -coordinates, not the y -coordinates. You can check each answer on your calculator: $\sin x$ and $\cos x$ should give the same value.
