

Transformations and Invariant Points

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

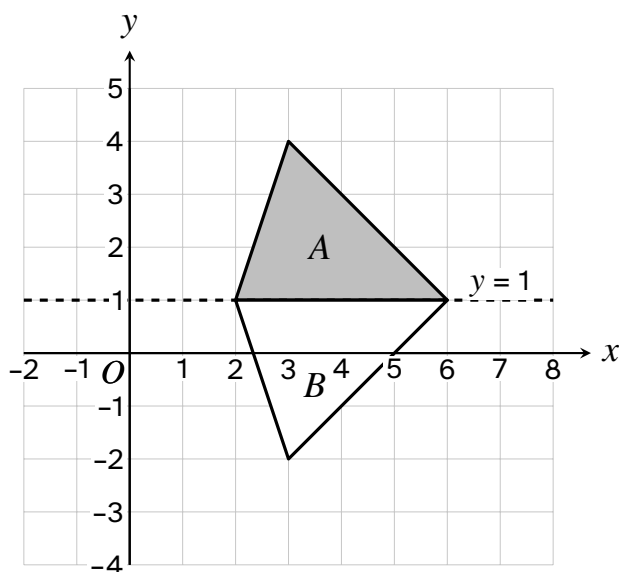
- (a) Reflect triangle *A* in the line $y = 1$. Label the image *B*.

Each point moves to the same distance on the other side of the line $y = 1$.

(3, 4) is 3 squares above the line, so it moves to 3 squares below: (3, -2).

(2, 1) and (6, 1) are on the line, so they stay where they are.

B: (2, 1), (6, 1), (3, -2)



Answer: (2, 1), (6, 1), (3, -2)

- (b) Write down the coordinates of the invariant points of this reflection that are vertices of the triangle.

An invariant point is a point that does not move.

In a reflection, the points on the mirror line do not move.

Two vertices of the triangle are on the line $y = 1$: $(2, 1)$ and $(6, 1)$.

Answer: $(2, 1)$ and $(6, 1)$

Tip: Check every vertex that touches the mirror line. There are two here, not one.

Marks: 1 mark for each correct point.

2 (1 mark)

Point P is invariant under a rotation of 90° clockwise about the origin.
Write down the coordinates of P .

In a rotation, every point turns around the centre.

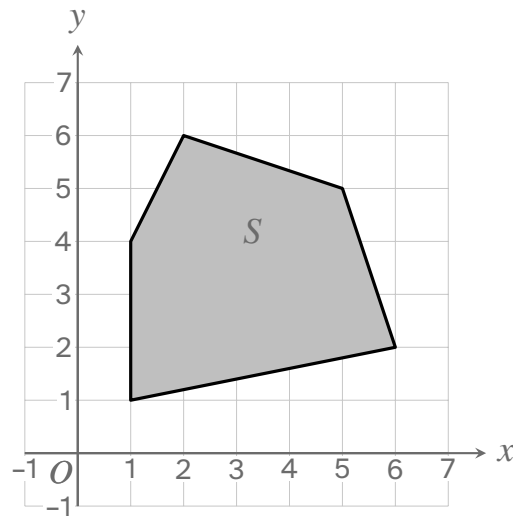
The only point that does not move is the centre of rotation itself.

The centre is the origin.

Answer: $(0, 0)$

3 (1 mark)

Shape S is shown on the grid.



S is reflected in the line $y = x$

Which of these vertices of S are invariant? Circle each one.

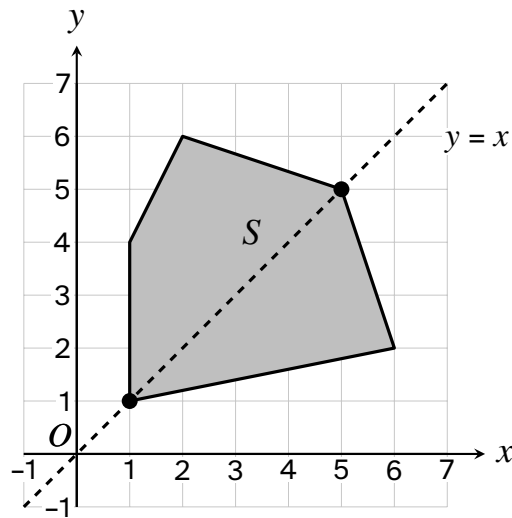
Options: (6, 2), (5, 5), (1, 1), (2, 6)

The invariant points are the points on the mirror line $y = x$.

These are the points where the x - and y -coordinates are equal.

(5, 5) and (1, 1) have $x = y$, so they are on the line.

(6, 2) and (2, 6) are not on the line: each one swaps with the other point, so they move.

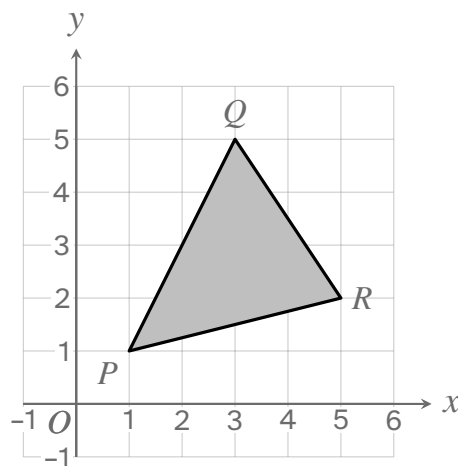


Answer: (5, 5) and (1, 1)

Tip: “Circle each one” tells you there may be more than one answer. The line $y = x$ is not drawn for you: it goes through (0, 0), (1, 1), (2, 2), and so on.

4 (1 mark)

Triangle PQR is shown on the grid.



PQR is reflected in the line $y = 2x - 1$

Which side of the triangle is invariant? Circle your answer.

Options: PQ , QR , PR , none of them

A side is invariant if both of its ends are on the mirror line $y = 2x - 1$.

$P(1, 1)$: $2 \times 1 - 1 = 1$, so P is on the line ✓

$Q(3, 5)$: $2 \times 3 - 1 = 5$, so Q is on the line ✓

$R(5, 2)$: $2 \times 5 - 1 = 9$, not 2, so R is not on the line.

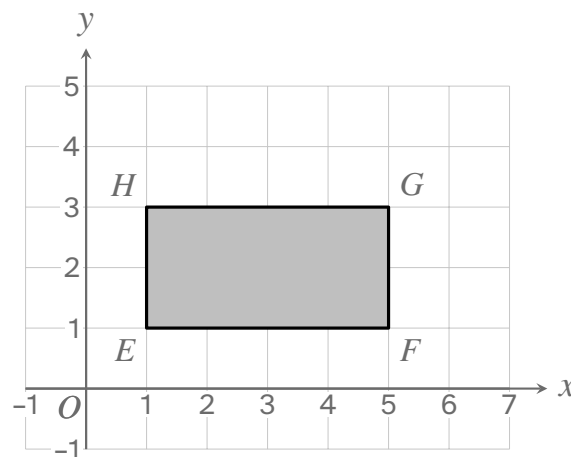
So side PQ lies on the mirror line, and every point on it is invariant.

Answer: PQ

Tip: Substitute the x -coordinate into $y = 2x - 1$ and compare with the y -coordinate. In a reflection, the invariant points are exactly the points on the mirror line.

5 (3 marks)

$EFGH$ is a rectangle.



(a) $EFGH$ is reflected in the line $y = 2$

How many points on the perimeter of $EFGH$ are invariant? Circle your answer.

Options: 0, 1, 2, infinitely many

Points on the mirror line $y = 2$ do not move.

The line $y = 2$ crosses the rectangle halfway up.

It meets the perimeter at $(1, 2)$ and $(5, 2)$ only.

Answer: 2

Tip: The line $y = 2$ goes across the inside of the rectangle, but only the two points where it crosses the sides are on the perimeter.

- (b) $EFGH$ is reflected in the line $x = 5$

How many points on the perimeter of $EFGH$ are invariant? Circle your answer.

Options: 0, 1, 2, infinitely many

Points on the mirror line $x = 5$ do not move.

The side FG lies on the line $x = 5$.

Every point on FG stays where it is, so there are infinitely many.

Answer: infinitely many

- (c) $EFGH$ is rotated 180° about $(3, 2)$

How many points on the perimeter of $EFGH$ are invariant? Circle your answer.

Options: 0, 1, 2, infinitely many

In a rotation, only the centre does not move.

$(3, 2)$ is the centre of the rectangle, which is inside it, not on the perimeter.

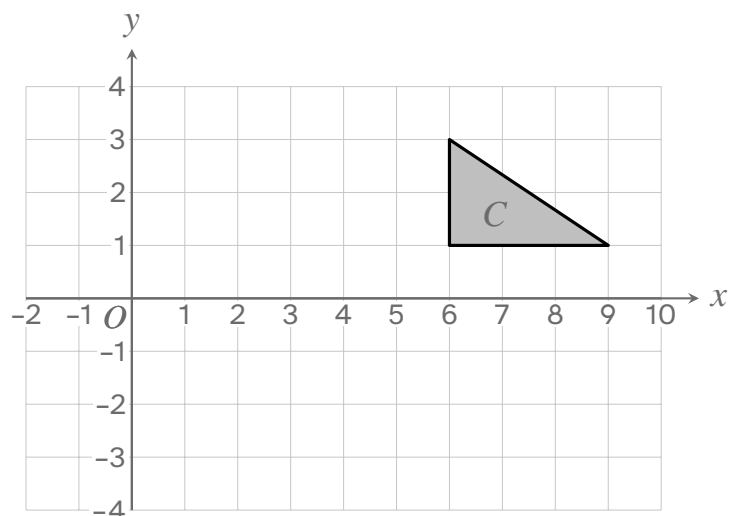
The rectangle maps onto itself, but every point on the perimeter moves (e.g. E moves to G).

Answer: 0

Tip: The rectangle lands on top of itself, but that does not make its points invariant. Each point must stay in exactly the same place.

6 (2 marks)

Triangle C is shown on the grid.



Triangle C is rotated 180° about $(5, 0)$, and the image is then reflected in the line $y = 0$. Describe fully the single transformation equivalent to these two transformations.

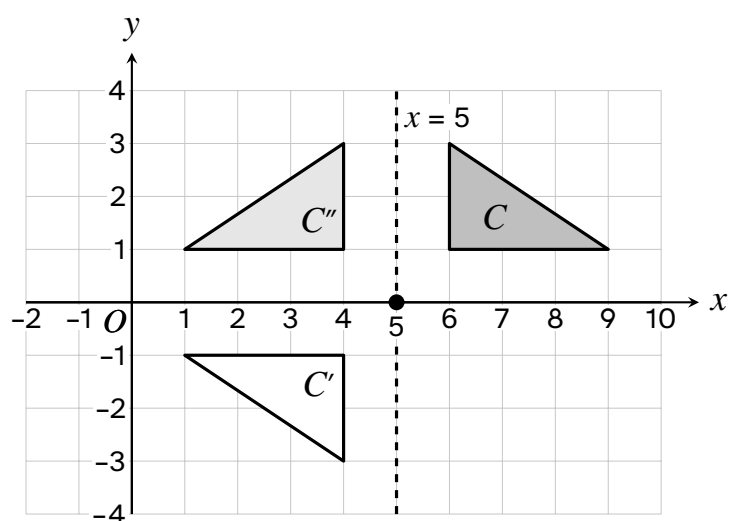
Do each transformation on the grid.

Rotate C by 180° about $(5, 0)$ to give C' . The vertices

$(6, 1), (9, 1), (6, 3)$ go to $(4, -1), (1, -1), (4, -3)$

Reflect C' in $y = 0$ (the x -axis) to give C'' . Its vertices are

$(4, 1), (1, 1), (4, 3)$



C'' is the mirror image of C in the vertical line halfway between them.

Check: $(6, 1) \rightarrow (4, 1)$ and $(9, 1) \rightarrow (1, 1)$; halfway between 6 and 4 is 5, and halfway between 9 and 1 is 5.

This is a reflection in the line $x = 5$.

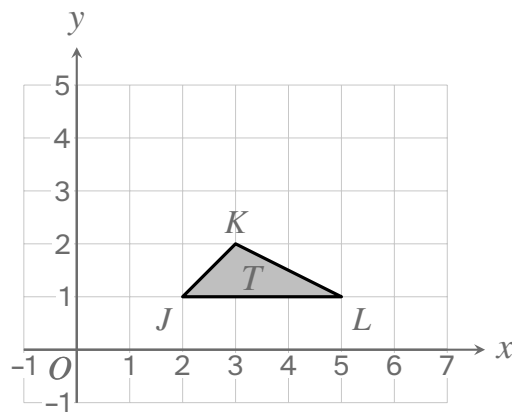
Answer: Reflection in the line $x = 5$

Tip: Draw both images on the grid, then compare the final triangle with C . Use the word “reflection” (not “flip”) and give the equation of the mirror line.

Marks: 1 mark for “reflection”. 1 mark for the line $x = 5$.

7 (3 marks)

Triangle T is shown on the grid.



T is enlarged, scale factor 3

Vertex J is invariant under this enlargement.

(a) Work out the coordinates of the image of K .

J is invariant, so $J(2, 1)$ is the centre of enlargement.

From J to $K(3, 2)$: 1 right and 1 up.

Multiply by the scale factor 3: 3 right and 3 up.

$$(2 + 3, 1 + 3) = (5, 4)$$

Answer: (5, 4)

Tip: Measure from the centre, not from the origin. $(3 \times 3, 2 \times 3) = (9, 6)$ is a common wrong answer.

Marks: 1 mark for using J as the centre, or for 3 right and 3 up. 1 mark for (5, 4).

- (b) Explain why no other point on T is invariant.

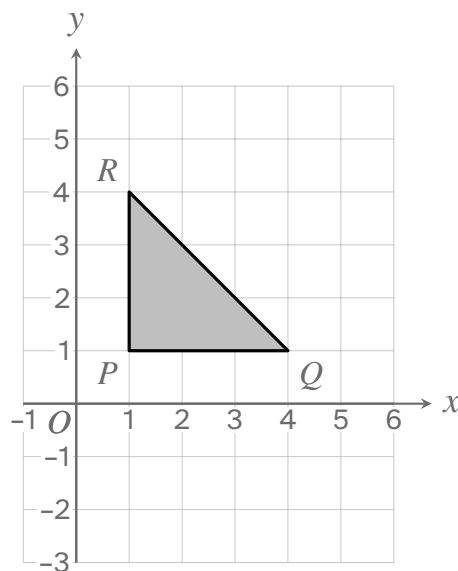
In an enlargement with scale factor 3, every point moves 3 times as far from the centre.

Only the centre itself (distance 0) stays where it is.

Answer: The only invariant point of an enlargement (scale factor not 1) is its centre, J . Every other point moves.

8 (5 marks)

Triangle PQR is shown on the grid.



- (a) A single transformation maps triangle PQR onto a new triangle. Q and R are invariant, and P moves to $(4, 4)$. Describe fully the transformation.

From the grid: $P(1, 1)$, $Q(4, 1)$, $R(1, 4)$

Q and R do not move, so the transformation is a reflection in the line through Q and R .

$$\text{Gradient of } QR = \frac{4-1}{1-4} = -1$$

$$y = -x + c \text{ through } Q(4,1): 1 = -4 + c, \text{ so } c = 5$$

$$\text{Mirror line: } y = -x + 5 \text{ (or } x + y = 5)$$

Check: the midpoint of $P(1,1)$ and $(4,4)$ is $(2.5, 2.5)$, and $2.5 + 2.5 = 5$ ✓

Answer: Reflection in the line $y = -x + 5$ (or $x + y = 5$)

Tip: Two invariant points that are not the same point tell you it is a reflection: the mirror line goes through both of them.

Marks: 1 mark for “reflection”. 1 mark for $y = -x + 5$ or $x + y = 5$.

- (b) Triangle PQR is rotated 90° clockwise about a point.

The image of R is $(4,1)$.

Find the centre of the rotation, and write down which vertex of PQR is invariant.

The centre is the same distance from $R(1,4)$ and its image $(4,1)$, and the turn is 90° clockwise.

Try $P(1,1)$: R is 3 up from P .

Turn “3 up” through 90° clockwise: it becomes 3 right.

3 right of P is $(4,1)$ ✓

So the centre is $(1,1)$.

The centre of a rotation is invariant, so vertex P is invariant.

Answer: Centre $(1,1)$; P is invariant

Tip: Tracing paper makes this quick: trace the triangle, pin a trial centre and turn the paper 90° clockwise.

Marks: 1 mark for a correct method (e.g. tracing paper, or turning the vector from a trial centre to R). 1 mark for $(1,1)$. 1 mark for P .

9 (5 marks)

Triangle ABC is drawn on the grid.

ABC is enlarged, scale factor 2, centre $(1,1)$, to give triangle P .

Triangle P is then translated by $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ to give triangle Q .

(a) Draw triangle P and triangle Q on the grid.

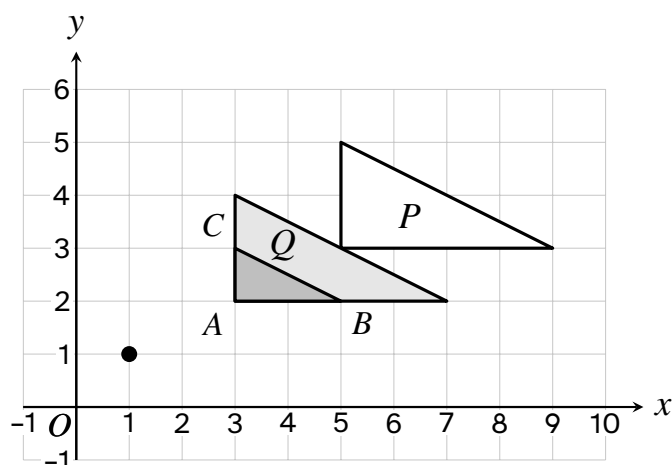
Enlarge from $(1,1)$: each vertex moves twice as far from $(1,1)$.

$A(3,2)$ is 2 right, 1 up from $(1,1)$, so it goes to 4 right, 2 up: $(5,3)$.

P : $(5,3), (9,3), (5,5)$

Then move every vertex 2 left and 1 down.

Q : $(3,2), (7,2), (3,4)$



Answer: P : $(5,3), (9,3), (5,5)$; Q : $(3,2), (7,2), (3,4)$

Marks: 1 mark for P correct. 1 mark for Q correct (follow-through from your P).

(b) One vertex of triangle ABC is invariant under the combined transformation.
Which vertex?

Compare Q with ABC .

A was at $(3,2)$ and Q has a vertex at $(3,2)$.

So A ends where it started.

Answer: A $(3,2)$

- (c) Describe fully the single transformation that maps triangle ABC onto triangle Q .

Q is twice the size of ABC , the same way up.

So it is an enlargement, scale factor 2.

A is invariant, so A is the centre.

Check: $B(5, 2)$ is 2 right of A ; its image $(7, 2)$ is 4 right of A ✓

Answer: Enlargement, scale factor 2, centre $(3, 2)$

Tip: Give one transformation only. The invariant point of an enlargement is its centre.

Marks: 1 mark for enlargement with scale factor 2. 1 mark for centre $(3, 2)$.
