

The Sine Rule

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Which of these is the sine rule for triangle ABC with sides a , b and c ?

Circle your answer.

Options: $\frac{a}{\sin A} = \frac{b}{\sin B}$, $\frac{a}{\sin B} = \frac{b}{\sin A}$, $\frac{\sin A}{a} = \frac{b}{\sin B}$, $\frac{\sin a}{A} = \frac{\sin b}{B}$

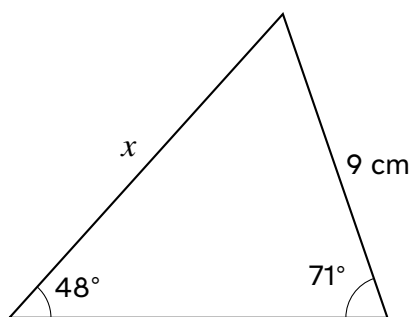
Side a is opposite angle A , and side b is opposite angle B .

In the sine rule, each side is divided by the sine of the angle **opposite** it.

Answer: $\frac{a}{\sin A} = \frac{b}{\sin B}$

2 (2 marks)

Here is a triangle.



Not drawn accurately

Use the sine rule to work out x .

x is opposite the 71° angle.

9 cm is opposite the 48° angle.

Sine rule: $\frac{x}{\sin 71^\circ} = \frac{9}{\sin 48^\circ}$

Multiply both sides by $\sin 71^\circ$:

$$x = \frac{9 \sin 71^\circ}{\sin 48^\circ} = 11.45 \dots$$

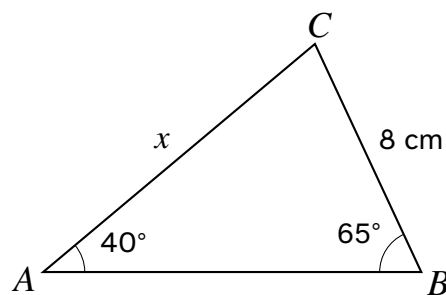
Answer: $x = 11.5$ cm (11.4 to 11.5 is fine)

Tip: Make sure your calculator is in degrees. Check for a small “D” at the top of the screen.

Marks: 1 mark for a correct sine rule equation. 1 mark for an answer from 11.4 to 11.5.

3 (3 marks)

Here is triangle ABC .



Not drawn accurately

Work out the length x .

Give your answer to 3 significant figures.

x is opposite the 65° angle.

8 cm is opposite the 40° angle.

Sine rule: $\frac{x}{\sin 65^\circ} = \frac{8}{\sin 40^\circ}$

Multiply both sides by $\sin 65^\circ$:

$$x = \frac{8 \sin 65^\circ}{\sin 40^\circ} = 11.279 \dots$$

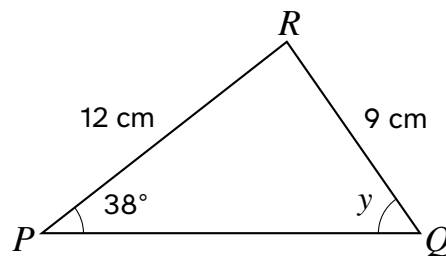
To 3 significant figures: 11.3

Answer: 11.3 cm

Marks: 1 mark for the sine rule. 1 mark for rearranging to $x = \frac{8 \sin 65^\circ}{\sin 40^\circ}$. 1 mark for 11.3.

4 (3 marks)

Here is triangle PQR .



Not drawn accurately

Angle y is acute.

Work out the size of angle y .

Give your answer to 1 decimal place.

y is opposite the 12 cm side.

38° is opposite the 9 cm side.

To find an angle, put the sines on top: $\frac{\sin y}{12} = \frac{\sin 38^\circ}{9}$

Multiply both sides by 12:

$$\sin y = \frac{12 \sin 38^\circ}{9} = 0.8209 \dots$$

$$y = \sin^{-1}(0.8209 \dots) = 55.17 \dots$$

y is acute, so this is the angle we want.

To 1 decimal place: 55.2°

Answer: 55.2°

Tip: Keep 0.8209 ... in your calculator. If you round it to 0.82 first, you get 55.1° , which is wrong.

Marks: 1 mark for the sine rule. 1 mark for $\sin y = \frac{12 \sin 38^\circ}{9}$. 1 mark for 55.2.

5 (4 marks)

In triangle XYZ , $XY = 15$ cm, angle $X = 48^\circ$ and angle $Y = 77^\circ$
Work out the length of YZ .

A sketch helps. Side XY is opposite angle Z .

Angles in a triangle add up to 180° :

$$\text{Angle } Z = 180 - 48 - 77 = 55^\circ$$

Side YZ is opposite angle $X = 48^\circ$.

$$\text{Sine rule: } \frac{YZ}{\sin 48^\circ} = \frac{15}{\sin 55^\circ}$$

$$YZ = \frac{15 \sin 48^\circ}{\sin 55^\circ} = 13.608 \dots$$

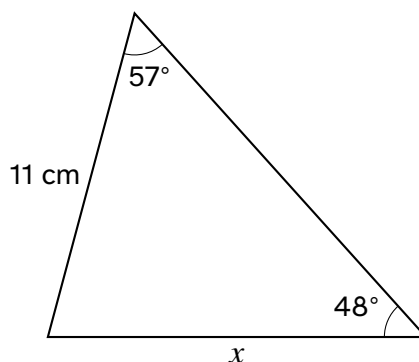
Answer: 13.6 cm

Tip: Each side must be paired with the angle opposite it. Find the missing angle first, because it is opposite the side you know.

Marks: 1 mark for angle $Z = 55^\circ$. 1 mark for the sine rule. 1 mark for rearranging. 1 mark for 13.6.

6 (1 mark)

Here is a triangle.



Not drawn accurately

Tomasz uses the sine rule to work out x .

Here is his working.

$$\frac{x}{\sin 57^\circ} = \frac{11}{\sin 48^\circ}$$
$$x = \frac{11 \sin 48^\circ}{\sin 57^\circ} = 9.75 \text{ cm}$$

His first line is correct.

What mistake has he made in his second line?

His first line pairs each side with the angle opposite it, so it is correct.

To get x on its own, multiply both sides by $\sin 57^\circ$.

$$\text{So } x = \frac{11 \sin 57^\circ}{\sin 48^\circ} = 12.4 \text{ cm}$$

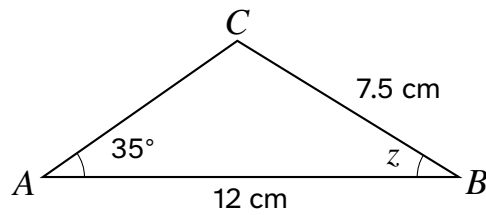
Tomasz swapped the two sines when he rearranged.

Answer: He rearranged wrongly: he should multiply by $\sin 57^\circ$, so $x = \frac{11 \sin 57^\circ}{\sin 48^\circ}$ (= 12.4 cm).

Tip: A quick check: the larger of two angles is opposite the longer side. $57^\circ > 48^\circ$, so x must be longer than 11 cm.

7 (3 marks)

Here is triangle ABC .



Not drawn accurately

Angle ACB is obtuse.

Work out the size of angle z .

Give your answer to 1 decimal place.

Angle C is opposite the 12 cm side. 35° is opposite the 7.5 cm side.

$$\text{Sine rule: } \frac{\sin C}{12} = \frac{\sin 35^\circ}{7.5}$$

$$\sin C = \frac{12 \sin 35^\circ}{7.5} = 0.9177 \dots$$

Your calculator gives $\sin^{-1}(0.9177 \dots) = 66.59 \dots^\circ$

But angle C is obtuse, so $C = 180 - 66.59 \dots = 113.40 \dots^\circ$

Angles in a triangle add up to 180° :

$$z = 180 - 35 - 113.40 \dots = 31.59 \dots$$

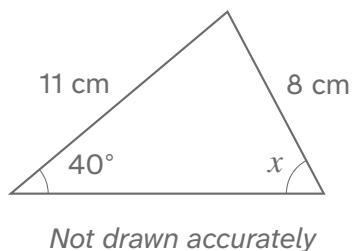
Answer: 31.6°

Tip: $\sin^{-1}$ on a calculator always gives an acute angle. Here angle C is obtuse, so use 180° minus your calculator answer. Using 66.6° gives $z = 78.4^\circ$, which is wrong.

Marks: 1 mark for $\sin C = \frac{12 \sin 35^\circ}{7.5}$. 1 mark for $180 - 66.6 (= 113.4)$. 1 mark for 31.6.

8 (4 marks)

(a) Here is triangle A.



Use the sine rule to show that $x = 62^\circ$ to the nearest degree.

x is opposite the 11 cm side.

40° is opposite the 8 cm side.

$$\text{Sine rule: } \frac{\sin x}{11} = \frac{\sin 40^\circ}{8}$$

$$\sin x = \frac{11 \sin 40^\circ}{8} = 0.8838 \dots$$

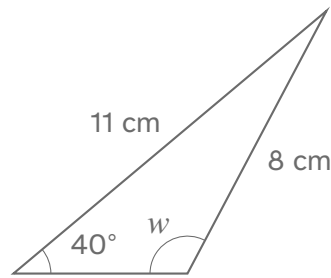
$$x = \sin^{-1}(0.8838 \dots) = 62.10 \dots$$

So $x = 62^\circ$ to the nearest degree ✓

Tip: In a “show that”, write down the more accurate value (62.10 ...) before rounding. Do not use 62° in your working.

Marks: 1 mark for the sine rule. 1 mark for $\sin x = 0.883 \dots$ 1 mark for seeing 62.1 (more accurate than 62).

(b) Here is triangle B .



Not drawn accurately

Anna thinks that w must be 62° to the nearest degree.

She says,

“This is because triangle B has two sides and one angle the same as triangle A .”

Without further calculation, is she correct?

Tick a box.

Give a reason for your answer.

The 40° angle is not between the 8 cm and 11 cm sides.

So two different triangles can be drawn with these measurements.

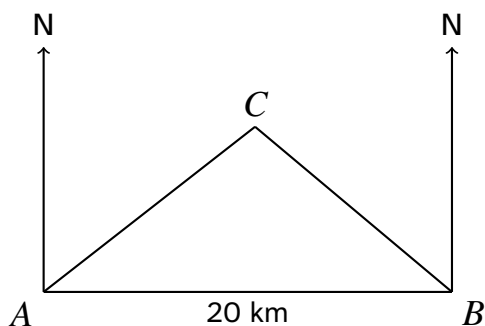
In triangle B , w is obtuse.

$$\sin(180^\circ - 62^\circ) = \sin 62^\circ, \text{ so } w = 180 - 62 = 118^\circ$$

Answer: No. The 40° angle is not between the two sides, so w can be obtuse:
 $w = 118^\circ$.

9 (4 marks)

The diagram shows the positions of A , B and C .



Not drawn accurately

Two coastguard stations, A and B , are 20 km apart.

B is due East of A .

A boat, C , is on a bearing of 052° from A and on a bearing of 310° from B .

Work out the distance of the boat from B .

Bearings are measured clockwise from North.

AB points due East, which is a bearing of 090° .

So angle $CAB = 90 - 52 = 38^\circ$

From B , due West (A) is a bearing of 270° .

So angle $CBA = 310 - 270 = 40^\circ$

Angle $ACB = 180 - 38 - 40 = 102^\circ$ (angles in a triangle)

BC is opposite the 38° angle, and 20 km is opposite 102° .

Sine rule: $\frac{BC}{\sin 38^\circ} = \frac{20}{\sin 102^\circ}$

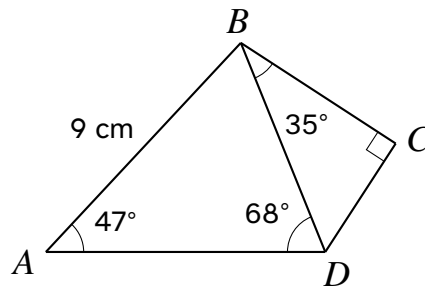
$$BC = \frac{20 \sin 38^\circ}{\sin 102^\circ} = 12.588 \dots$$

Answer: 12.6 km

Marks: 1 mark for 38° and 40° . 1 mark for 102° . 1 mark for the sine rule. 1 mark for 12.6.

10 (4 marks)

Here is a diagram.



Not drawn accurately

The diagram shows triangle ABD and a right-angled triangle BCD .

$AB = 9$ cm, angle $BAD = 47^\circ$ and angle $ADB = 68^\circ$

Angle $BCD = 90^\circ$ and angle $DBC = 35^\circ$

Work out the length of BC .

Give your answer to 3 significant figures.

First find BD in triangle ABD , which has no right angle.

BD is opposite the 47° angle, and $AB = 9$ is opposite the 68° angle.

Sine rule: $\frac{BD}{\sin 47^\circ} = \frac{9}{\sin 68^\circ}$

$$BD = \frac{9 \sin 47^\circ}{\sin 68^\circ} = 7.099 \dots \text{ cm}$$

Now use the right-angled triangle BCD . BD is the hypotenuse and BC is adjacent to 35° .

$$BC = 7.099 \dots \times \cos 35^\circ = 5.815 \dots$$

To 3 significant figures: 5.82

Answer: 5.82 cm

Tip: Keep $BD = 7.099 \dots$ in your calculator for the next step. The sine rule is needed in triangle ABD ; SOHCAHTOA only works in the right-angled triangle BCD .

Marks: 1 mark for the sine rule in triangle ABD . 1 mark for $BD = 7.10$. 1 mark for $BC = \text{your } BD \times \cos 35^\circ$. 1 mark for 5.82.