

The Quadratic Formula

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Solve $x^2 + 4x - 7 = 0$

Give your answers to 2 decimal places.

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here $a = 1$, $b = 4$ and $c = -7$.

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-7)}}{2 \times 1}$$

$$b^2 - 4ac = 16 + 28 = 44$$

$$\text{So } x = \frac{-4 \pm \sqrt{44}}{2}$$

$$\text{Using } +: x = \frac{-4 + \sqrt{44}}{2} = 1.316 \dots$$

$$\text{Using } -: x = \frac{-4 - \sqrt{44}}{2} = -5.316 \dots$$

Answer: $x = 1.32$ and $x = -5.32$

Tip: Take the sign with the number: here $c = -7$, not 7. Using $c = 7$ is the most common mistake in these questions.

Marks: 1 mark for correct substitution into the formula. 1 mark for 1.32. 1 mark for -5.32.

2 (3 marks)

Solve $2x^2 - 5x - 1 = 0$

Give your answers to 2 decimal places.

Here $a = 2$, $b = -5$ and $c = -1$.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$b^2 - 4ac = 25 + 8 = 33$$

$$\text{So } x = \frac{5 \pm \sqrt{33}}{4}$$

$$\text{Using } +: x = \frac{5 + \sqrt{33}}{4} = 2.686 \dots$$

$$\text{Using } -: x = \frac{5 - \sqrt{33}}{4} = -0.186 \dots$$

Answer: $x = 2.69$ and $x = -0.19$

Tip: The whole of the top, $5 \pm \sqrt{33}$, is divided by 4. On a calculator, put the top in brackets or use the fraction key.

Marks: 1 mark for correct substitution into the formula. 1 mark for 2.69. 1 mark for -0.19.

3 (3 marks)

Solve $x^2 - 6x + 4 = 0$

Give your answers in the form $a \pm \sqrt{b}$

Here $a = 1$, $b = -6$ and $c = 4$.

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 4}}{2 \times 1} = \frac{6 \pm \sqrt{20}}{2}$$

Simplify the surd: $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

$$\text{So } x = \frac{6 \pm 2\sqrt{5}}{2}$$

Divide both terms on the top by 2: $x = 3 \pm \sqrt{5}$

Answer: $x = 3 \pm \sqrt{5}$

Marks: 1 mark for correct substitution into the formula. 1 mark for $\sqrt{20} = 2\sqrt{5}$. 1 mark for the correct answer.

4 (4 marks)

Here is an equation.

$$(x + 3)(2x - 1) = 4x + 2$$

- (a) Write the equation in the form $ax^2 + bx + c = 0$ where a , b and c are integers.

Expand the brackets: $(x + 3)(2x - 1) = 2x^2 - x + 6x - 3$

$$= 2x^2 + 5x - 3$$

$$\text{So } 2x^2 + 5x - 3 = 4x + 2$$

Subtract $4x$ and subtract 2 from both sides:

$$2x^2 + x - 5 = 0$$

Answer: $2x^2 + x - 5 = 0$

Tip: Expand the brackets carefully ($+6x - x = +5x$), then move **every** term to one side so the other side is 0.

Marks: 1 mark for expanding the brackets correctly ($2x^2 + 5x - 3$). 1 mark for $2x^2 + x - 5 = 0$.

- (b) Hence, solve the equation.
Give your answers to 2 decimal places.

Use the formula with $a = 2$, $b = 1$ and $c = -5$:

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times (-5)}}{2 \times 2}$$

$$b^2 - 4ac = 1 + 40 = 41$$

$$\text{So } x = \frac{-1 \pm \sqrt{41}}{4}$$

Using +: $x = \frac{-1 + \sqrt{41}}{4} = 1.350 \dots$

Using -: $x = \frac{-1 - \sqrt{41}}{4} = -1.850 \dots$

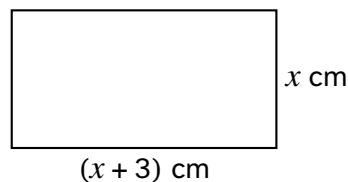
Answer: $x = 1.35$ and $x = -1.85$

Tip: Here $c = -5$, so $-4ac = -4 \times 2 \times (-5) = +40$. Getting this sign wrong gives $\sqrt{-39}$, which you cannot work out.

Marks: 1 mark for correct substitution into the formula, or for $\frac{-1 \pm \sqrt{41}}{4}$. 1 mark for both answers.

5 (5 marks)

The area of this rectangle is 20 cm^2 .



Not drawn accurately

- (a) Show that $x^2 + 3x - 20 = 0$

Area of a rectangle = length $\times$ width

So $x(x + 3) = 20$

Expand the bracket: $x^2 + 3x = 20$

Subtract 20 from both sides: $x^2 + 3x - 20 = 0$ as required.

Answer: Shown

Marks: 1 mark for $x(x + 3) = 20$. 1 mark for expanding and rearranging correctly.

- (b) Work out the width of the rectangle.
Give your answer to 2 decimal places.

Use the formula with $a = 1$, $b = 3$ and $c = -20$:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-20)}}{2 \times 1} = \frac{-3 \pm \sqrt{89}}{2}$$

$$x = 3.216 \dots \text{ or } x = -6.216 \dots$$

A length cannot be negative, so $x = 3.216 \dots$

The width is x cm.

Answer: 3.22 cm

Tip: The formula gives two solutions, but only the positive one can be a length. Say why you reject the negative one.

Marks: 1 mark for correct substitution into the formula. 1 mark for 3.216.... 1 mark for 3.22 with the negative solution rejected.

6 (4 marks)

Solve $3x^2 = 8x + 5$

Give your answers to 2 decimal places.

First rearrange so that one side is 0:

$$3x^2 - 8x - 5 = 0$$

So $a = 3$, $b = -8$ and $c = -5$.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 3 \times (-5)}}{2 \times 3}$$

$$b^2 - 4ac = 64 + 60 = 124$$

$$\text{So } x = \frac{8 \pm \sqrt{124}}{6}$$

$$x = 3.189... \text{ or } x = -0.522...$$

Answer: $x = 3.19$ and $x = -0.52$

Tip: Always rearrange to "... = 0" before you use the formula. Then put negative numbers in brackets: $(-8)^2 = 64$, but -8^2 on a calculator gives -64 .

Marks: 1 mark for $3x^2 - 8x - 5 = 0$. 1 mark for correct substitution into the formula. 1 mark for $\frac{8 \pm \sqrt{124}}{6}$. 1 mark for both answers.

7 (6 marks)

Solve $\frac{2x}{x+3} + \frac{x-1}{2x} = 2$

Give your solutions to 2 decimal places.

You must show your working.

Clear the fractions by multiplying every term by $2x(x+3)$:

$$\frac{2x}{x+3} \times 2x(x+3) = 4x^2$$

$$\frac{x-1}{2x} \times 2x(x+3) = (x-1)(x+3)$$

$$2 \times 2x(x+3) = 4x(x+3)$$

$$\text{So } 4x^2 + (x-1)(x+3) = 4x(x+3)$$

$$\text{Expand: } 4x^2 + x^2 + 2x - 3 = 4x^2 + 12x$$

$$\text{Simplify: } 5x^2 + 2x - 3 = 4x^2 + 12x$$

$$\text{Rearrange so one side is 0: } x^2 - 10x - 3 = 0$$

Use the formula with $a = 1$, $b = -10$ and $c = -3$:

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4 \times 1 \times (-3)}}{2} = \frac{10 \pm \sqrt{112}}{2}$$

$$x = 10.291... \text{ or } x = -0.291...$$

Answer: $x = 10.29$ and $x = -0.29$

Tip: Multiply every term by $2x(x + 3)$, including the 2 on the right. A common error is to multiply only the fractions, which gives the wrong quadratic.

Marks: 1 mark for multiplying by $2x(x + 3)$ (or writing the left side as one fraction). 1 mark for expanding correctly. 1 mark for $x^2 - 10x - 3 = 0$. 1 mark for correct substitution into the formula. 1 mark for 10.29. 1 mark for -0.29 .
