

The Cosine Rule

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

In triangle ABC , side a is opposite angle A .

Which of these is equal to a^2 ?

Circle your answer.

Options: $b^2 + c^2 - 2bc \cos A$, $b^2 + c^2 + 2bc \cos A$, $b^2 - c^2 - 2bc \cos A$

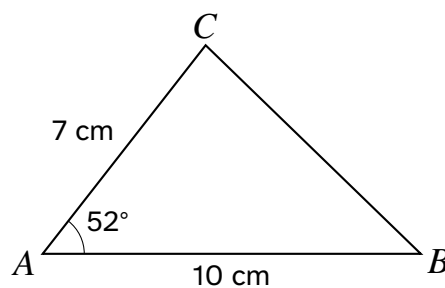
The cosine rule is $a^2 = b^2 + c^2 - 2bc \cos A$

The two squares are added, and $2bc \cos A$ is subtracted.

Answer: $b^2 + c^2 - 2bc \cos A$

2 (3 marks)

Here is triangle ABC .



Not drawn accurately

Work out the length BC .

Give your answer to 3 significant figures.

We know two sides (7 cm and 10 cm) and the angle between them (52°).

So use the cosine rule to find the third side.

$$BC^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 52^\circ$$

$$BC^2 = 149 - 86.19 \dots = 62.80 \dots$$

$$\text{Square root: } BC = \sqrt{62.80 \dots} = 7.925 \dots$$

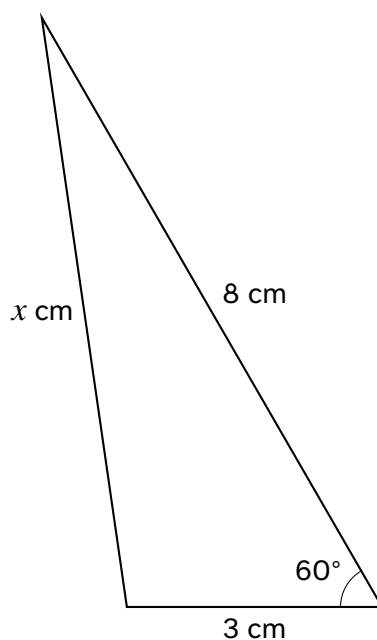
Answer: 7.93 cm

Tip: Type the whole right-hand side into your calculator in one go. Working out $149 - 140$ first and then multiplying by $\cos 52^\circ$ is a common mistake.

Marks: 1 mark for the correct substitution. 1 mark for the square root of your value. 1 mark for 7.93.

3 (3 marks)

Here is a triangle.



Not drawn accurately

$$\cos 60^\circ = 0.5$$

Use the cosine rule to show that $x = 7$

You must show your working.

x is opposite the 60° angle, between the 3 cm and 8 cm sides.

$$x^2 = 3^2 + 8^2 - 2 \times 3 \times 8 \times \cos 60^\circ$$

$$x^2 = 9 + 64 - 48 \times 0.5$$

$$x^2 = 73 - 24 = 49$$

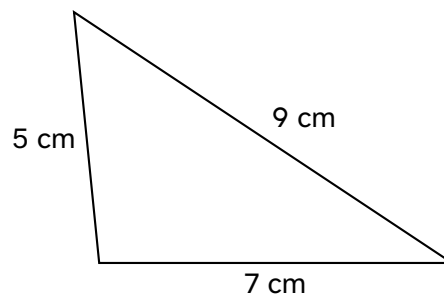
$$x = \sqrt{49} = 7 \quad \checkmark$$

Tip: In a “show that”, write every line, including $x^2 = 49$ and $x = \sqrt{49}$. Work out 48×0.5 before you subtract.

Marks: 1 mark for the correct substitution. 1 mark for $73 - 24$ or 49 . 1 mark for $x = 7$ with all the working shown.

4 (3 marks)

A triangle has sides 5 cm, 7 cm and 9 cm.



Not drawn accurately

Work out the size of the largest angle.

Give your answer to 1 decimal place.

The largest angle is opposite the longest side, 9 cm.

To find an angle, use the rearranged cosine rule:

$$\cos \theta = \frac{b^2 + c^2 - a^2}{2bc}, \text{ where } a = 9 \text{ is the side opposite } \theta.$$

$$\cos \theta = \frac{5^2 + 7^2 - 9^2}{2 \times 5 \times 7} = \frac{-7}{70} = -0.1$$

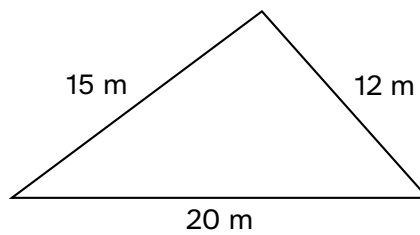
$$\theta = \cos^{-1}(-0.1) = 95.73 \dots$$

Answer: 95.7°

Marks: 1 mark for the correct substitution. 1 mark for $\cos \theta = -0.1$. 1 mark for 95.7.

5 (3 marks)

A garden is in the shape of a triangle with sides 12 m, 15 m and 20 m.



Not drawn accurately

Work out the size of the smallest angle of the garden.
Give your answer to 1 decimal place.

The smallest angle is opposite the shortest side, 12 m.

$$\cos \theta = \frac{15^2 + 20^2 - 12^2}{2 \times 15 \times 20}$$

$$\cos \theta = \frac{225 + 400 - 144}{600} = \frac{481}{600} = 0.8016 \dots$$

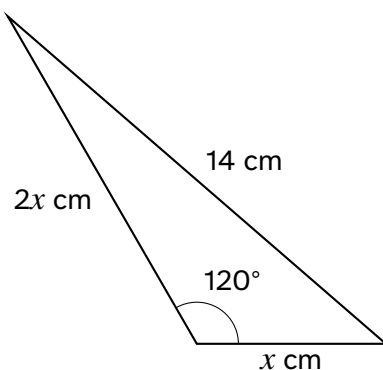
$$\theta = \cos^{-1}(0.8016 \dots) = 36.71 \dots$$

Answer: 36.7°

Marks: 1 mark for the correct substitution. 1 mark for $\cos \theta = \frac{481}{600}$. 1 mark for 36.7.

6 (3 marks)

Here is a triangle.



Not drawn accurately

$$\cos 120^\circ = -0.5$$

Work out the value of x .

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

14 cm is opposite the 120° angle.

$$14^2 = x^2 + (2x)^2 - 2 \times x \times 2x \times \cos 120^\circ$$

$$(2x)^2 = 4x^2 \text{ and } 2 \times x \times 2x \times (-0.5) = -2x^2$$

$$196 = x^2 + 4x^2 + 2x^2$$

$$196 = 7x^2$$

$$x^2 = 28$$

$$x = \sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$$

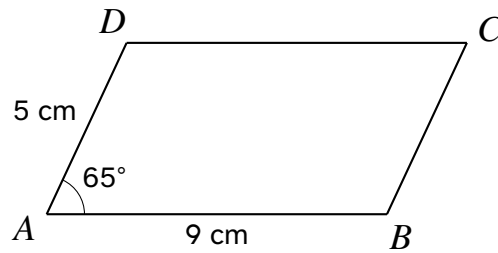
Answer: $x = 2\sqrt{7}$

Tip: Take care with the signs: $-2 \times x \times 2x \times (-0.5) = +2x^2$. Also, $(2x)^2 = 4x^2$, not $2x^2$.

Marks: 1 mark for a correct cosine rule equation. 1 mark for $7x^2 = 196$. 1 mark for $2\sqrt{7}$.

7 (4 marks)

Here is a parallelogram.



Not drawn accurately

$ABCD$ is a parallelogram.

Work out the length of the longer diagonal of the parallelogram.

Co-interior angles add up to 180° (AD is parallel to BC).

So angle $ABC = 180 - 65 = 115^\circ$

The longer diagonal is opposite the larger angle, so it is AC .

In triangle ABC : $AB = 9$, $BC = 5$ (opposite sides of a parallelogram are equal).

$$AC^2 = 9^2 + 5^2 - 2 \times 9 \times 5 \times \cos 115^\circ$$

$$AC^2 = 106 + 38.03 \dots = 144.03 \dots$$

$$AC = \sqrt{144.03 \dots} = 12.00 \dots$$

Answer: 12.0 cm

Tip: Using 65° gives the shorter diagonal (BD). The longer diagonal is opposite the obtuse angle.

Marks: 1 mark for 115° . 1 mark for the cosine rule with 115° . 1 mark for the square root. 1 mark for 12.0.

8 (4 marks)

Two boats leave a harbour at the same time.

Boat P sails 40 km on a bearing of 070°

Boat Q sails 55 km on a bearing of 195°

Work out the distance between the boats.

Draw a sketch with a North line at the harbour, H .

Both bearings are measured from the same North line.

So angle $PHQ = 195 - 70 = 125^\circ$

$$PQ^2 = 40^2 + 55^2 - 2 \times 40 \times 55 \times \cos 125^\circ$$

$$PQ^2 = 4625 + 2523.7 \dots = 7148.7 \dots$$

$$PQ = \sqrt{7148.7 \dots} = 84.55 \dots$$

Answer: 84.6 km

Marks: 1 mark for 125° . 1 mark for the cosine rule. 1 mark for the square root. 1 mark for 84.6.

9 (4 marks)

Maya cycles from her home, H , to a café, C .

C is 7.2 km from H on a bearing of 300°

She then cycles 5.5 km from C to a lake, L , on a bearing of 045°

Maya then cycles in a straight line from L back to H .

Work out the total distance Maya cycles.

Draw a sketch with North lines at H and C .

The bearing of H from C is the back bearing: $300 - 180 = 120^\circ$

L is on a bearing of 045° from C .

So angle $HCL = 120 - 45 = 75^\circ$

$$HL^2 = 7.2^2 + 5.5^2 - 2 \times 7.2 \times 5.5 \times \cos 75^\circ$$

$$HL^2 = 82.09 - 20.49 \dots = 61.59 \dots$$

$$HL = \sqrt{61.59 \dots} = 7.848 \dots \text{ km}$$

$$\text{Total distance} = 7.2 + 5.5 + 7.848 \dots = 20.548 \dots$$

= 20.5 km (3 s.f.)

Answer: 20.5 km

Tip: For a bearing more than 180° , the back bearing is $300 - 180 = 120^\circ$, not $300 + 180$. A sketch with both North lines makes the angle at C clear.

Marks: 1 mark for angle $HCL = 75^\circ$. 1 mark for the cosine rule with 75° . 1 mark for $HL = 7.85$. 1 mark for 20.5.
