

Surds

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the expression equal to $\sqrt{50}$

Options: $25\sqrt{2}$, $5\sqrt{2}$, $2\sqrt{5}$, $10\sqrt{5}$

Find a square number that is a factor of 50: $50 = 25 \times 2$

$$\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

Answer: $5\sqrt{2}$

2 (5 marks)

Simplify fully

(a) $\sqrt{12} + \sqrt{27}$

Simplify each surd using a square factor:

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

$$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$$

$$\text{Now add like terms: } 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$

Answer: $5\sqrt{3}$

Tip: $\sqrt{12} + \sqrt{27}$ is **not** $\sqrt{39}$. You can only add surds once they have the same number under the root.

Marks: 1 mark for $2\sqrt{3}$ or $3\sqrt{3}$. 1 mark for $5\sqrt{3}$.

(b) $\sqrt{45} - \sqrt{20}$

$$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$$

$$\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

$$3\sqrt{5} - 2\sqrt{5} = \sqrt{5}$$

Answer: $\sqrt{5}$

Marks: 1 mark for $3\sqrt{5}$ or $2\sqrt{5}$. 1 mark for $\sqrt{5}$.

(c) $\sqrt{8} \times \sqrt{18}$

Use $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$$\sqrt{8} \times \sqrt{18} = \sqrt{144} = 12$$

Answer: 12

Tip: $\sqrt{8} \times \sqrt{18}$ is $\sqrt{144}$, which is 12, not 144.

3 (2 marks)

A rectangle has length $\sqrt{12}$ cm and width $\sqrt{8}$ cm.

Show that the area of the rectangle is $4\sqrt{6}$ cm².

$$\text{Area} = \text{length} \times \text{width} = \sqrt{12} \times \sqrt{8}$$

$$= \sqrt{96}$$

96 = 16 × 6, and 16 is a square number.

$$\sqrt{96} = \sqrt{16} \times \sqrt{6} = 4\sqrt{6} \quad \checkmark$$

Answer: $\sqrt{96} = 4\sqrt{6}$

Marks: 1 mark for $\sqrt{96}$ (or $2\sqrt{3} \times 2\sqrt{2}$). 1 mark for showing it equals $4\sqrt{6}$.

4 (3 marks)

Simplify fully $\sqrt{75} + \sqrt{108} - \sqrt{48}$

Each number is 3 times a square number.

$$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$$

$$\sqrt{108} = \sqrt{36} \times \sqrt{3} = 6\sqrt{3}$$

$$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$

$$\text{Collect like terms: } 5\sqrt{3} + 6\sqrt{3} - 4\sqrt{3} = 7\sqrt{3}$$

Answer: $7\sqrt{3}$

Tip: Use the **largest** square factor. $\sqrt{108} = \sqrt{4} \times \sqrt{27} = 2\sqrt{27}$ is not fully simplified, because 27 still has the square factor 9.

Marks: 1 mark for one of $5\sqrt{3}$, $6\sqrt{3}$, $4\sqrt{3}$. 1 mark for all three. 1 mark for $7\sqrt{3}$.

5 (3 marks)

The edges of a cuboid have lengths $\sqrt{10}$ cm, $3\sqrt{2}$ cm and $\sqrt{20}$ cm.

Work out the volume of the cuboid.

Give your answer as an integer.

$$\text{Volume} = \text{length} \times \text{width} \times \text{height} = \sqrt{10} \times 3\sqrt{2} \times \sqrt{20}$$

$$\text{Multiply the surds: } \sqrt{10} \times \sqrt{2} \times \sqrt{20} = \sqrt{10 \times 2 \times 20} = \sqrt{400}$$

$$\sqrt{400} = 20$$

$$\text{Now multiply by the 3: } 3 \times 20 = 60$$

$$(\text{Another way: } \sqrt{10} \times \sqrt{20} = \sqrt{200} = 10\sqrt{2}, \text{ then } 10\sqrt{2} \times 3\sqrt{2} = 30 \times 2 = 60)$$

Answer: 60 cm^3

Tip: Keep the 3 in front of $\sqrt{2}$ separate. Multiply the numbers under the roots together, then multiply by 3 at the end.

Marks: 1 mark for multiplying the three lengths. 1 mark for $\sqrt{400}$ or $10\sqrt{2}$ (or $\sqrt{3600}$). 1 mark for 60.

6 (4 marks)

Expand and simplify

(a) $(3 + \sqrt{2})(5 - \sqrt{2})$

Multiply each term in the first bracket by each term in the second:

$$3 \times 5 = 15$$

$$3 \times (-\sqrt{2}) = -3\sqrt{2}$$

$$\sqrt{2} \times 5 = 5\sqrt{2}$$

$$\sqrt{2} \times (-\sqrt{2}) = -2$$

$$\text{Collect like terms: } 15 - 2 - 3\sqrt{2} + 5\sqrt{2} = 13 + 2\sqrt{2}$$

Answer: $13 + 2\sqrt{2}$

Marks: 1 mark for all four terms correct. 1 mark for $13 + 2\sqrt{2}$.

(b) $(\sqrt{5} + 2)^2$

Write it as two brackets: $(\sqrt{5} + 2)(\sqrt{5} + 2)$

$$\sqrt{5} \times \sqrt{5} = 5$$

$$\sqrt{5} \times 2 = 2\sqrt{5} \text{ (twice)}$$

$$2 \times 2 = 4$$

$$\text{Total: } 5 + 2\sqrt{5} + 2\sqrt{5} + 4 = 9 + 4\sqrt{5}$$

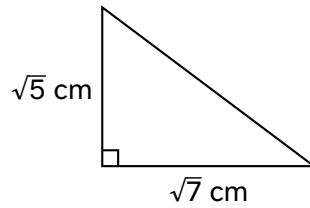
Answer: $9 + 4\sqrt{5}$

Tip: $(\sqrt{5} + 2)^2$ is not $5 + 4$. Write out both brackets so you do not miss the two middle terms.

Marks: 1 mark for all four terms correct. 1 mark for $9 + 4\sqrt{5}$.

7 (3 marks)

Here is a right-angled triangle.



Not drawn accurately

Work out the length of the hypotenuse.

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

Use Pythagoras: $\text{hypotenuse}^2 = (\sqrt{5})^2 + (\sqrt{7})^2$

Squaring a square root gives back the number: $(\sqrt{5})^2 = 5$ and $(\sqrt{7})^2 = 7$

$$\text{hypotenuse}^2 = 5 + 7 = 12$$

$$\text{hypotenuse} = \sqrt{12}$$

$$\text{Simplify: } \sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

Answer: $2\sqrt{3}$ cm

Tip: $(\sqrt{5})^2 = 5$, not 25 and not 2.5. Squaring undoes the square root.

Marks: 1 mark for using Pythagoras. 1 mark for $\sqrt{12}$. 1 mark for $2\sqrt{3}$.

8 (3 marks)

$$(2\sqrt{3} + a)^2 = b + 8\sqrt{3}$$

a and b are integers.

Work out the values of a and b .

$$\text{Expand: } (2\sqrt{3} + a)(2\sqrt{3} + a)$$

$$2\sqrt{3} \times 2\sqrt{3} = 4 \times 3 = 12$$

$$\text{Middle terms: } 2a\sqrt{3} + 2a\sqrt{3} = 4a\sqrt{3}$$

$$a \times a = a^2$$

$$\text{So } (2\sqrt{3} + a)^2 = 12 + a^2 + 4a\sqrt{3}$$

Compare the $\sqrt{3}$ terms: $4a = 8$, so $a = 2$

Compare the whole numbers: $b = 12 + a^2 = 12 + 4 = 16$

Answer: $a = 2$, $b = 16$

Marks: 1 mark for expanding the bracket correctly. 1 mark for $4a = 8$ (or $b = 12 + a^2$). 1 mark for $a = 2$ and $b = 16$.
