

Spheres, Cones, Pyramids and Frustums

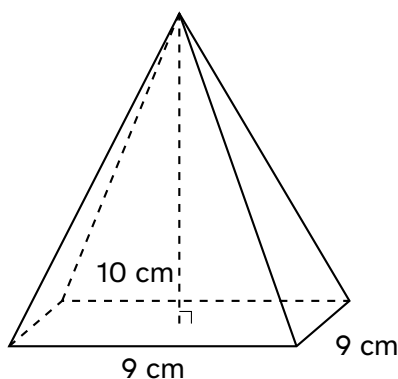
Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Here is a square-based pyramid.



Not drawn accurately

The base is a square of side 9 cm.

The perpendicular height of the pyramid is 10 cm.

Work out the volume of the pyramid.

Volume of a pyramid = $\frac{1}{3} \times \text{area of base} \times \text{height}$

Area of base = $9 \times 9 = 81 \text{ cm}^2$

$$\text{Volume} = \frac{1}{3} \times 81 \times 10 = 270$$

Answer: 270 cm^3

Tip: Use the **area** of the square base (9×9), not just the side length 9.

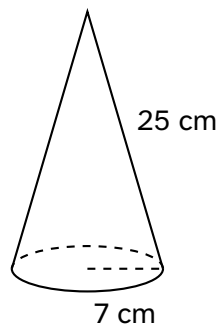
Marks: 1 mark for $\frac{1}{3} \times 81 \times 10$. 1 mark for 270.

2 (5 marks)

Here is a solid cone.

The radius of the base is 7 cm.

The slant height is 25 cm.



Not drawn accurately

- (a) Show that the total surface area of the cone is $224\pi \text{ cm}^2$

$$\text{Curved surface} = \pi r l = \pi \times 7 \times 25 = 175\pi$$

$$\text{The base is a circle: } \pi \times 7^2 = 49\pi$$

$$\text{Total} = 175\pi + 49\pi = 224\pi$$

Answer: $224\pi \text{ cm}^2$

Tip: “Total” surface area includes the circular base as well as the curved surface.

Marks: 1 mark for 175π or 49π . 1 mark for 224π .

- (b) Work out the volume of the cone.
Give your answer in terms of π .

The volume needs the perpendicular height h , not the slant height.

The radius, height and slant height make a right-angled triangle.

$$h = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} = 24 \text{ cm}$$

$$\text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 7^2 \times 24$$

$$= \frac{1}{3} \times 1176\pi = 392\pi$$

Answer: $392\pi \text{ cm}^3$

Tip: Do not use the slant height 25 in the volume formula. Use Pythagoras to find the perpendicular height first, and subtract the squares.

Marks: 1 mark for $h = 24$. 1 mark for $\frac{1}{3}\pi \times 49 \times 24$. 1 mark for 392π .

3 (2 marks)

A sphere has volume 500 cm^3 .

Work out its radius.

Give your answer to 3 significant figures.

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\frac{4}{3} \pi r^3 = 500$$

$$\text{Multiply by 3 and divide by } 4\pi: r^3 = \frac{1500}{4\pi} = 119.3 \dots$$

$$r = \sqrt[3]{119.3 \dots} = 4.923 \dots$$

$$= 4.92 \text{ (3 s.f.)}$$

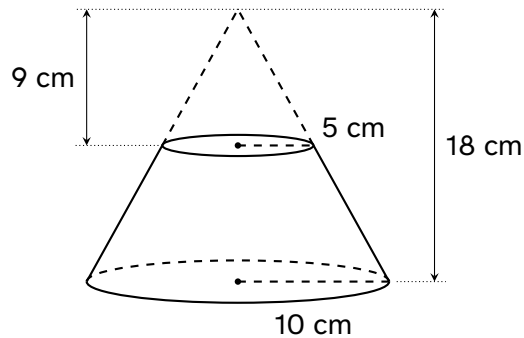
Answer: 4.92 cm

Marks: 1 mark for $r^3 = \frac{3 \times 500}{4\pi}$. 1 mark for 4.92.

4 (3 marks)

The diagram shows a frustum.

It is a cone of radius 10 cm and height 18 cm with a cone of radius 5 cm and height 9 cm removed from the top.



Not drawn accurately

Work out the volume of the frustum.

Give your answer in terms of π .

Frustum = large cone – small cone

$$\text{Large cone: } \frac{1}{3}\pi \times 10^2 \times 18 = 600\pi$$

$$\text{Small cone: } \frac{1}{3}\pi \times 5^2 \times 9 = 75\pi$$

$$\text{Frustum} = 600\pi - 75\pi = 525\pi$$

Answer: $525\pi \text{ cm}^3$

Tip: Halving the radius and the height does not halve the volume. Work out both cones in full, then subtract.

Marks: 1 mark for the volume of either cone. 1 mark for subtracting. 1 mark for 525π .

5 (3 marks)

A solid is made from a cylinder of radius 3 cm and height 10 cm with a hemisphere of radius 3 cm on top.

Work out the volume of the solid.

Give your answer to 3 significant figures.

$$\text{Cylinder: } \pi r^2 h = \pi \times 3^2 \times 10 = 90\pi$$

A hemisphere is half a sphere.

$$\text{Hemisphere: } \frac{1}{2} \times \frac{4}{3} \pi \times 3^3 = \frac{1}{2} \times 36\pi = 18\pi$$

$$\text{Total} = 90\pi + 18\pi = 108\pi = 339.2 \dots$$

$$= 339 \text{ (3 s.f.)}$$

Answer: 339 cm^3

Tip: Halve the sphere's volume once for a hemisphere. Using the whole sphere, or halving twice, are both common mistakes.

Marks: 1 mark for 90π . 1 mark for 18π . 1 mark for 339.

6 (4 marks)

A container is a hollow cone with its vertex at the bottom.

The top of the cone is a horizontal circle of radius 8 cm.

The height of the cone is 20 cm.

Water is poured into the empty container at a constant rate of 20 cm^3 per second.

How long does it take for the depth of the water to reach 15 cm?

Give your answer to the nearest second.

The water forms a cone with its vertex at the bottom, similar to the container.

$$\text{Scale factor} = \frac{15}{20} = \frac{3}{4}$$

$$\text{Radius of the water surface} = 8 \times \frac{3}{4} = 6 \text{ cm}$$

$$\text{Volume of water} = \frac{1}{3} \pi \times 6^2 \times 15 = 180\pi = 565.48 \dots \text{ cm}^3$$

$$\text{Time} = 565.48 \dots \div 20 = 28.27 \dots \text{ seconds}$$

= 28 seconds (to the nearest second)

Answer: 28 seconds

Tip: The water is a smaller cone, so its radius is not 8 cm. Use similar shapes: the radius and the height shrink by the same scale factor.

Marks: 1 mark for radius 6. 1 mark for $\frac{1}{3}\pi \times 6^2 \times 15$. 1 mark for 180π or 565.5. 1 mark for 28.

7 (3 marks)

A cylinder has radius r and height h .

A hemisphere has radius r .

The volume of the cylinder is equal to the volume of the hemisphere.

Work out the ratio $r:h$

You must show your working.

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Volume of hemisphere} = \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3$$

$$\text{They are equal: } \pi r^2 h = \frac{2}{3} \pi r^3$$

$$\text{Divide both sides by } \pi r^2: h = \frac{2}{3} r$$

$$r:h = r:\frac{2}{3}r$$

$$\text{Multiply both sides by 3 and divide by } r: r:h = 3:2$$

Answer: 3 : 2

Tip: Check the order: the question asks for $r:h$, so 2:3 would be wrong.

Marks: 1 mark for $\pi r^2 h = \frac{2}{3} \pi r^3$. 1 mark for $h = \frac{2}{3} r$. 1 mark for 3:2.

8 (3 marks)

A solid cone has base radius $3x$ and slant height $5x$.

A sphere has radius r .

The total surface area of the cone is equal to the surface area of the sphere.

Work out r in terms of x .

Give your answer in its simplest form.

$$\text{Cone, curved surface: } \pi r l = \pi \times 3x \times 5x = 15\pi x^2$$

$$\text{Cone, circular base: } \pi (3x)^2 = 9\pi x^2$$

$$\text{Cone total} = 15\pi x^2 + 9\pi x^2 = 24\pi x^2$$

$$\text{Sphere: } 4\pi r^2$$

$$4\pi r^2 = 24\pi x^2$$

$$\text{Divide both sides by } 4\pi: r^2 = 6x^2$$

$$r = \sqrt{6}x \text{ (a length is positive)}$$

Answer: $r = \sqrt{6}x$

Tip: $(3x)^2 = 9x^2$, not $3x^2$. When you square root $6x^2$, the answer is $\sqrt{6}x$, not $6x$.

Marks: 1 mark for $24\pi x^2$. 1 mark for $4\pi r^2 = 24\pi x^2$. 1 mark for $\sqrt{6}x$.

9 (4 marks)

A solid sphere fits exactly inside a hollow cube.

The sphere touches all six faces of the cube.

What percentage of the space inside the cube is **not** filled by the sphere?

Give your answer to 1 decimal place.

You must show your working.

Let the radius of the sphere be r .

The sphere touches opposite faces, so the side of the cube is the diameter, $2r$.

$$\text{Volume of cube} = (2r)^3 = 8r^3$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Fraction filled} = \frac{\frac{4}{3}\pi r^3}{8r^3} = \frac{\pi}{6} = 0.5235 \dots$$

Fraction not filled = $1 - 0.5235 \dots = 0.4764 \dots$

= 47.6%

Answer: 47.6%

Tip: The side of the cube is the **diameter** of the sphere, $2r$. So the cube's volume is $8r^3$, not r^3 or $2r^3$.

Marks: 1 mark for the side $2r$ (or choosing a value for r and doubling it). 1 mark for $8r^3$ and $\frac{4}{3}\pi r^3$. 1 mark for $\frac{\pi}{6}$ or 52.4%. 1 mark for 47.6.
