

Special Sequences - Higher

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Which of these is a geometric progression?

Circle your answer.

Options: 1, 4, 9, 16, 2, 5, 8, 11, 5, 10, 20, 40, 1, 1, 2, 3

In a geometric progression you multiply by the same number each time.

5, 10, 20, 40: each term is the one before $\times 2$.

(1, 4, 9, 16 are square numbers, 2, 5, 8, 11 adds 3 each time and 1, 1, 2, 3 is a Fibonacci-type sequence.)

Answer: 5, 10, 20, 40

Tip: Geometric means **multiply** by the same number each time. Arithmetic means **add** the same number each time.

2 (2 marks)

In a Fibonacci-type sequence, each term after the first two is the sum of the two terms before it.

The 1st term is 6 and the 3rd term is -1

Work out the 2nd term and the 4th term.

1st term + 2nd term = 3rd term

So $6 + 2\text{nd term} = -1$

2nd term = $-1 - 6 = -7$

4th term = 2nd term + 3rd term

= $-7 + (-1) = -8$

Check: 6, -7, -1, -8 and $6 + (-7) = -1$ ✓

Answer: 2nd term -7, 4th term -8

Tip: Work backwards carefully: the 2nd term is the 3rd term **minus** the 1st term, $-1 - 6 = -7$, not -5.

Marks: 1 mark for -7. 1 mark for -8 (or a correct follow-through from your 2nd term).

3 (2 marks)

Here is a geometric progression.

3 6 12 24 ...

(a) Write down the common ratio.

The common ratio is what you multiply by each time.

$$6 \div 3 = 2$$

Answer: 2

(b) Work out the 8th term.

Keep multiplying by 2:

3, 6, 12, 24, 48, 96, 192, 384

(Or: 8th term = $3 \times 2^7 = 3 \times 128 = 384$)

Answer: 384

4 (2 marks)

A geometric progression has 1st term 2 and 3rd term 18

Every term in the progression is positive.

Work out the sum of the first four terms.

Let the common ratio be r .

From the 1st term to the 3rd term you multiply by r twice, which is r^2 .

$$r^2 = 18 \div 2 = 9$$

All the terms are positive, so $r = 3$.

The first four terms are 2, 6, 18, 54

$$\text{Sum} = 2 + 6 + 18 + 54 = 80$$

Answer: 80

Tip: From the 1st term to the 3rd term is **two** steps, so divide to get r^2 , not r .

Marks: 1 mark for $r = 3$ or the 2nd term 6. 1 mark for 80.

5 (2 marks)

The first two terms of a geometric progression are $\sqrt{2}$ and 2
Work out the 5th term.

$$\text{Common ratio} = 2 \div \sqrt{2} = \sqrt{2}$$

$$(\text{Check: } \sqrt{2} \times \sqrt{2} = 2 \quad \checkmark)$$

Keep multiplying by $\sqrt{2}$:

$$\sqrt{2}, 2, 2\sqrt{2}, 4, 4\sqrt{2}$$

Answer: $4\sqrt{2}$

Tip: Remember $\sqrt{2} \times \sqrt{2} = 2$. So $2\sqrt{2} \times \sqrt{2} = 2 \times 2 = 4$.

Marks: 1 mark for the common ratio $\sqrt{2}$. 1 mark for $4\sqrt{2}$.

6 (2 marks)

The n th term of a geometric progression is $48 \times \left(\frac{1}{2}\right)^n$
Work out the first term of the progression that is less than 1
Give your answer as a fraction.

Substitute $n = 1, 2, 3, \dots$

$$48 \times \frac{1}{2} = 24, \quad 48 \times \frac{1}{4} = 12, \quad 48 \times \frac{1}{8} = 6$$

$$48 \times \frac{1}{16} = 3, \quad 48 \times \frac{1}{32} = \frac{48}{32} = 1\frac{1}{2}$$

$$48 \times \frac{1}{64} = \frac{48}{64} = \frac{3}{4}$$

$\frac{3}{4}$ is the first term less than 1 (it is the 6th term).

Answer: $\frac{3}{4}$

Tip: Each term is half of the one before, so you can keep halving: 24, 12, 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$.

Marks: 1 mark for working out terms up to $1\frac{1}{2}$ or for $\frac{48}{64}$. 1 mark for $\frac{3}{4}$.

7 (3 marks)

The n th term of a sequence is $(1 + \sqrt{2})^n$

Show that the third term is $7 + 5\sqrt{2}$

The third term is $(1 + \sqrt{2})^3$.

First square the bracket:

$$(1 + \sqrt{2})^2 = 1 + \sqrt{2} + \sqrt{2} + 2 = 3 + 2\sqrt{2}$$

Now multiply by $(1 + \sqrt{2})$ again:

$$(3 + 2\sqrt{2})(1 + \sqrt{2}) = 3 + 3\sqrt{2} + 2\sqrt{2} + 2 \times 2$$

$$= 3 + 5\sqrt{2} + 4$$

$$= 7 + 5\sqrt{2}$$

Answer: $(1 + \sqrt{2})^3 = 7 + 5\sqrt{2}$

Marks: 1 mark for expanding two brackets. 1 mark for $3 + 2\sqrt{2}$. 1 mark for $7 + 5\sqrt{2}$ with correct working.

8 (3 marks)

In a Fibonacci-type sequence, each term after the first two is the sum of the two terms before it.

The 1st term is 3 and the 6th term is 44

Work out the 2nd term.

Let the 2nd term be k .

The terms are 3, k , $3 + k$, $3 + 2k$, $6 + 3k$, $9 + 5k$

So the 6th term is $9 + 5k$

$$9 + 5k = 44$$

$$\text{Subtract 9: } 5k = 35$$

$$\text{Divide by 5: } k = 7$$

Check: 3, 7, 10, 17, 27, 44 ✓

Answer: 7

Tip: Write each term in terms of k , collecting like terms as you go. Then form an equation using the 6th term.

Marks: 1 mark for 3rd and 4th terms in terms of k . 1 mark for $9 + 5k$ or $9 + 5k = 44$. 1 mark for 7.

9 (4 marks)

An arithmetic progression and a geometric progression both have 1st term 3

The two progressions also have the same 2nd term.

The 3rd term of the arithmetic progression is 15

Work out the difference between the 4th term of the geometric progression and the 4th term of the arithmetic progression.

Arithmetic: the 1st term is 3 and the 3rd term is 15, so two common differences make $15 - 3 = 12$

$$\text{Common difference} = 12 \div 2 = 6$$

Arithmetic progression: 3, 9, 15, 21

The geometric progression also has 2nd term 9, so its common ratio is $9 \div 3 = 3$

Geometric progression: 3, 9, 27, 81

Difference between the 4th terms: $81 - 21 = 60$

Answer: 60

Tip: Don't assume the 3rd terms are equal: only the 1st and 2nd terms are the same, so the geometric progression's 3rd term is 27, not 15.

Marks: 1 mark for the common difference 6 (or 2nd term 9). 1 mark for the 4th term of the arithmetic progression, 21. 1 mark for the common ratio 3 or the 4th term 81. 1 mark for 60.

10 (3 marks)

The n th triangular number is $\frac{n(n+1)}{2}$

Prove that 8 times any triangular number, plus 1, is always a square number.

8 times the n th triangular number, plus 1, is $8 \times \frac{n(n+1)}{2} + 1$

$8 \div 2 = 4$, so this is $4n(n+1) + 1$

Expand: $4n^2 + 4n + 1$

Factorise: $(2n+1)^2$

$2n+1$ is a whole number, so $(2n+1)^2$ is always a square number.

Answer: $(2n+1)^2$, which is always a square number.

Marks: 1 mark for $4n(n+1) + 1$. 1 mark for $4n^2 + 4n + 1$. 1 mark for $(2n+1)^2$ with a conclusion.
