

Simplifying Algebraic Fractions

Solutions

For AQA GCSE Mathematics (8300) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Simplify fully

(a) $\frac{6x^2y}{9xy^3}$

Simplify the numbers: $\frac{6}{9} = \frac{2}{3}$

Simplify the x terms: $\frac{x^2}{x} = x$

Simplify the y terms: $\frac{y}{y^3} = \frac{1}{y^2}$

Put these together.

Answer: $\frac{2x}{3y^2}$

Marks: 2 marks for the correct answer. 1 mark if two of the three parts (number, x , y) are correct.

(b) $\frac{4x + 12}{x^2 + 3x}$

Factorise the top: $4x + 12 = 4(x + 3)$

Factorise the bottom: $x^2 + 3x = x(x + 3)$

So the fraction is $\frac{4(x + 3)}{x(x + 3)}$

Cancel the common factor $(x + 3)$.

Answer: $\frac{4}{x}$

Tip: You can only cancel factors (things that are multiplied), not terms (things that are added). Factorise first, then cancel whole brackets.

Marks: 1 mark for factorising the top and the bottom. 1 mark for $\frac{4}{x}$.

2 (1 mark)

Circle the expression equal to $\frac{x^2 - 4x}{x - 4}$

Options: $-4x$, x , $x - 1$, x^2

Factorise the top: $x^2 - 4x = x(x - 4)$

So the fraction is $\frac{x(x - 4)}{x - 4}$

Cancel $(x - 4)$ from the top and the bottom.

Answer: x

3 (3 marks)

Simplify fully $\frac{x^2 - 9}{x^2 + x - 6}$

Top: this is a difference of two squares.

$$x^2 - 9 = (x + 3)(x - 3)$$

$$\text{Bottom: } x^2 + x - 6 = (x + 3)(x - 2)$$

So the fraction is $\frac{(x + 3)(x - 3)}{(x + 3)(x - 2)}$

Cancel the common factor $(x + 3)$.

Answer: $\frac{x - 3}{x - 2}$

Tip: Don't cancel the x^2 terms and then tidy up what is left. You must factorise the top and the bottom first.

Marks: 1 mark for factorising the top. 1 mark for factorising the bottom. 1 mark for $\frac{x - 3}{x - 2}$.

4 (3 marks)

Simplify fully $\frac{2x^2 + 5x - 3}{4x^2 - 1}$

Top: $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

Bottom: this is a difference of two squares, because $4x^2 = (2x)^2$ and $1 = 1^2$.

$$4x^2 - 1 = (2x - 1)(2x + 1)$$

So the fraction is $\frac{(2x - 1)(x + 3)}{(2x - 1)(2x + 1)}$

Cancel the common factor $(2x - 1)$.

Answer: $\frac{x + 3}{2x + 1}$

Marks: 1 mark for factorising the top. 1 mark for factorising the bottom. 1 mark for $\frac{x + 3}{2x + 1}$.

5 (3 marks)

Simplify fully $\frac{6x - 9x^2}{15x - 10}$

Top: take out the common factor $3x$.

$$6x - 9x^2 = 3x(2 - 3x)$$

Bottom: take out the common factor 5.

$$15x - 10 = 5(3x - 2)$$

The brackets $(2 - 3x)$ and $(3x - 2)$ are not the same, but $2 - 3x = -(3x - 2)$.

So the top is $-3x(3x - 2)$

The fraction is $\frac{-3x(3x - 2)}{5(3x - 2)}$

Cancel the common factor $(3x - 2)$.

Answer: $-\frac{3x}{5}$

Tip: $(2 - 3x)$ and $(3x - 2)$ do not cancel to give 1. They cancel to give -1 , because $2 - 3x = -(3x - 2)$.

Marks: 1 mark for $3x(2 - 3x)$ or $-3x(3x - 2)$. 1 mark for $5(3x - 2)$. 1 mark for $-\frac{3x}{5}$.

6 (5 marks)

(a) Factorise $2x^2 + 7x - 15$

Multiply the first and last numbers: $2 \times (-15) = -30$

Find two numbers that multiply to -30 and add to 7: 10 and -3

Split the middle term:

$$2x^2 + 10x - 3x - 15$$

Factorise in pairs:

$$2x(x + 5) - 3(x + 5)$$

$$= (2x - 3)(x + 5)$$

Answer: $(2x - 3)(x + 5)$

Marks: 1 mark for a correct method, e.g. $2x(x + 5) - 3(x + 5)$. 1 mark for $(2x - 3)(x + 5)$.

(b) Simplify fully $\frac{2x^2 + 7x - 15}{x^2 - 25}$

Top: use part (a). $2x^2 + 7x - 15 = (2x - 3)(x + 5)$

Bottom: this is a difference of two squares.

$$x^2 - 25 = (x + 5)(x - 5)$$

So the fraction is $\frac{(2x-3)(x+5)}{(x+5)(x-5)}$

Cancel the common factor $(x+5)$.

Answer: $\frac{2x-3}{x-5}$

Marks: 1 mark for $(x+5)(x-5)$. 1 mark for using part (a) and cancelling $(x+5)$. 1 mark for $\frac{2x-3}{x-5}$.

7 (3 marks)

Simplify fully $\frac{x^4 - 9x^2}{2x + 6}$

Top: take out the common factor x^2 .

$$x^4 - 9x^2 = x^2(x^2 - 9)$$

$x^2 - 9$ is a difference of two squares: $(x+3)(x-3)$

So the top is $x^2(x+3)(x-3)$

Bottom: $2x + 6 = 2(x+3)$

The fraction is $\frac{x^2(x+3)(x-3)}{2(x+3)}$

Cancel the common factor $(x+3)$.

Answer: $\frac{x^2(x-3)}{2}$

Tip: After taking out x^2 , look again: $x^2 - 9$ factorises too. Keep going until nothing else will cancel.

Marks: 1 mark for $x^2(x^2 - 9)$. 1 mark for $(x+3)(x-3)$. 1 mark for the correct answer, or an equivalent such as $\frac{x^3 - 3x^2}{2}$.

8 (3 marks)

Ali says,

“For $x \neq 3$, $\frac{2x^2 - 18}{x - 3}$ simplifies to $2x - 6$ ”

Is Ali correct?

You must show your working.

Top: take out the common factor 2.

$$2x^2 - 18 = 2(x^2 - 9)$$

$x^2 - 9$ is a difference of two squares: $(x + 3)(x - 3)$

The fraction is $\frac{2(x + 3)(x - 3)}{x - 3}$

Cancel $(x - 3)$: $2(x + 3) = 2x + 6$

So the fraction simplifies to $2x + 6$, not $2x - 6$. Ali is not correct.

Answer: No, it simplifies to $2x + 6$

Tip: $x^2 - 9$ factorises to $(x + 3)(x - 3)$. After cancelling $(x - 3)$, the bracket that is left is $(x + 3)$, so the sign is +.

Marks: 1 mark for $2(x^2 - 9)$. 1 mark for $(x + 3)(x - 3)$. 1 mark for $2x + 6$ and No.

9 (3 marks)

a and b are positive values.

Show that $\frac{4a^2 - b^2}{2a^2 + ab}$ can be written as $2 - \frac{b}{a}$

Top: this is a difference of two squares, because $4a^2 = (2a)^2$.

$$4a^2 - b^2 = (2a + b)(2a - b)$$

Bottom: take out the common factor a : $2a^2 + ab = a(2a + b)$

So the fraction is $\frac{(2a + b)(2a - b)}{a(2a + b)}$

Cancel the common factor $(2a + b)$: $\frac{2a - b}{a}$

Split into two fractions: $\frac{2a}{a} - \frac{b}{a} = 2 - \frac{b}{a}$

Answer: $\frac{2a - b}{a} = 2 - \frac{b}{a}$

Tip: Look for a difference of two squares with two letters: $4a^2 - b^2 = (2a)^2 - b^2$.

Marks: 1 mark for $(2a + b)(2a - b)$. 1 mark for $a(2a + b)$. 1 mark for a full correct method reaching $2 - \frac{b}{a}$.
